

Talent competition:
the interrelationship between individual competencies and industry
structure and evolution

by

M.A. Carree

Faculty of Economics and Business Administration
Maastricht University
and
Faculty of Economics
Erasmus University Rotterdam

W. van Olffen

Faculty of Economics and Business Administration
Maastricht University

Preliminary version, May 2001

JEL-codes: J24, L11

Keywords: Talent distribution, Industry evolution, Industry structure, Employment

Correspondence (e-mail): m.carree@mw.unimaas.nl

M.A. Carree
MW-ORG
Faculty of Economics and Business Administration
Maastricht University
PO Box 616
6200 MD Maastricht
The Netherlands

ABSTRACT

The key sources of a firm's performance are the capabilities of the entrepreneur (CEO) and the employees, and the extent to which these are made productive. In many service industries the competition for the productive capabilities of individuals (so-called talents) is a basic determinant of the evolution of the industry structure. In our model firms consist of entrepreneurs hiring individuals that match their own combination of talents (firm-level social reproduction). Individuals choose companies in which their specific combination of talents are best valued. We explore the (comparative) statics of this model, plus its dynamic properties when employees learn and when technological change causes the relative importance of certain talents over others to shift over time. The findings are translated to a list of testable propositions that may guide future empirical work.

1. Introduction

A rich and interrelated literature has developed in the fields of organizational demographic evolution on the one hand and the evolution of industries and population ecology on the other.¹ The focus of debate in these literatures is how organizations and groups of related organizations evolve over time. The main subject of population ecological research has been the evolution of the *number* of (market) participants, i.e. the entry, survival, exit and merging of (market) players. Industry evolution research has usually focused on a much broader spectrum of variables including developments of demand, price, unit cost, market shares, R&D and innovation rates. Industries are composed of organizations that themselves also evolve as workers (especially top managers) enter and leave or experience up- and downward mobility within the organization. The origin of these intra-organizational developments has been the subject of research in the organizational demographic evolution literature.

Although the two evolutionary processes, at the industry and organizational levels, can be interrelated in various ways, there has as yet been little research *connecting* these processes of internal and external organizational developments. The influence can run both ways. Earlier work in population ecology has shown an influence of population-level phenomena like birth, dissolution and mergers on demographic dynamics like job mobility and movement of employees into and out of firms (Carroll, Haveman and Swaminathan, 1992; Haveman and Cohen, 1994; Haveman, 1995). However, for the reverse causal effect, that is: organizational demographics influencing industry-level variables, some interesting path-breaking has started only fairly recently as population ecology theorists shifted their attention more explicitly toward issues of organizational demography (Sørensen, 1999; Carroll and Hannan, 2000). For instance, Sørensen (1999) shows that organizations possessing similar distributions of tenure to other organizations in the industry, are engaged in greater competition than organizations that do not share this important demographic characteristic. Thus, using ecological crowding theory, intra-organizational demographics influence both inter-organizational processes and industry-level outcomes. In the present paper we wish to build upon this latter strand of research, concentrating on a connection at an even more dissimilar level of aggregation, that of *individual* attributes and *industry structure* outcomes.

¹ For the field of organizational demographic evolution, see e.g. Pfeffer (1983) and Hambrick and Mason (1984). For literature on evolution of industries, see e.g. Klepper and Graddy (1990), Jovanovic and MacDonald (1994), Carree and Thurik (2000) and Klepper and Simons (2000a,b). For the field of population ecology, see e.g. Hannan and Freeman (1977) and Carroll and Hannan (2000).

We provide a radically *individualized* view on firm and industry dynamics, that we argue is highly descriptive of especially professional service industries, where workers' (given) capabilities are crucial for the performance of the firms. We develop a model relating the equilibrium industry structure outcome as determined by the workers' capabilities and their desire to achieve the maximum reward (profit or wage) by making them productive. Some workers who have high levels of productive capabilities (so-called talents) or a rare combination of various capabilities may find it most rewarding to start firms and be self-employed. These individuals may hire additional workers, who benefit to some extent from the employers' talents, to increase profit. Employers offer such workers a wage higher than their competitors or a potential level of profit in case the worker would start a new firm: it is a matter of *talent competition*. The choices of each of the workers between self-employment and employment at one of the incumbent firms determine the equilibrium market structure outcome.

We focus our attention on *productive individuals* migrating through and between organizations. We adopt a perspective in which workers are characterized by a multi-dimensional 'vector of talents', being more or less capable in various dimensions relevant for production. Competition for these resourceful people takes place both between and within organizations. In this perspective, competing successfully amounts to *attracting* the right people (from outside) and by internally *employing* them in the best possible way. The individuals, from their part, try to make the best of their capabilities by choosing either to join and switch organizations or to sell their produce on their own. As such, we take a different perspective on individual mobility than the traditional one in organization demography where individuals are regarded as rather passive entities simply 'filling jobs'. As Haveman (1995) points out, job mobility is usually treated in 'structuralist' terms where the primary influencing factors on job mobility are in the system, not in the individual. As she puts it (Haveman (1995: 589)): "Explaining organizational demography by reference to change in social structure requires a theoretical framework that is explicitly organizational" and, referring to White's (1970) 'vacancy-chain model': "jobs are viewed as relatively stable socio-economic positions, *independent of the individuals who occupy them*" (emphasis added). We feel this view reduces the individual too much to a pawn in a world of simple vacancy-filling companies. Instead, we will take a more individual perspective in which we endow people with a more self-conscious attitude, carrying a specific and stable mix of productive and sought-after talents which *they choose* to employ in firms that allow them to reap the highest possible benefits from such talents. The central questions that guide our effort

are twofold. First, we wonder if and how different individual productive endowments can affect the structure of the market (static viewpoint). Second, we explore how this market structure is affected by learning, technological change and creative destruction (dynamic viewpoint).

The intuition for a relation between processes of organizational and industry evolution is perhaps most visibly illustrated during periods of (technological) turmoil in which the required capabilities of organizations shift. As these capabilities importantly reside in the people employed and are relatively stable over time, this offers possibilities for people having the requisite characteristics to further their career *inside* an organization as their relative value increases. On the other hand it increases the competition for such people *between* organizations. For instance, the increasing importance of information technology (IT) throughout the 1980s and 1990s put a premium on people having IT skills and knowledge. While IT-‘talents’ in firms saw their career prospects rise, at the same time a huge external market for ‘IT-people’ (*sic*) developed on which companies competed for the most talented, bidding up their salaries in the process. This has prompted many IT-specialists not to be employed but to start their own firms. To the extent they chose to deploy their IT-talents in the same business, this boiled down to market entry. Thus, the increased importance of a certain talent squeezed out those talents to the open market resulting in intensified competition. It is an example of an adjustment in the distribution of productive talent to reach a more efficient equilibrium. Note that these adjustments in internal and external organization can be (partly) off-setting: if incumbents can adapt to the new technologies quickly enough by spotting, retaining and promoting talents or by developing them within the company,² there may not be much scope for new entrants. They can also be (partly) reinforcing: strong entry of the talented into the market may force incumbents to change their internal organization as fast as possible. They may do so by increasing career opportunities or pay to people holding the newly valued talents in order to keep them within the company.

In this paper we proceed as follows. In the next section we present the theoretical assumptions underlying the model and a general overview of its consequences. Section three describes the formal operationalization of the model and in section four we derive equilibrium properties of the model. In section five we use model simulations to explore the (comparative)

² Firms will show important short-run inertia in developing their employees' talents. Several reasons can be advanced for this. First, the relative inertia in workers' talents made them to choose for specific organizations, reducing the size of the talent pool. Second, the individuals most susceptible to training (i.e., those with *latent* talents) will have to be sought out, which takes time. Third, firms have difficulty changing in fundamental ways from prior experience and are at first uncertain about the exact content of new talents required.

static properties of the model. In section six, we move to a dynamic viewpoint and determine the effects on market structure of a technological change when introducing learning and technological progress and when altering the relative importance of individuals' talents. Finally, in section seven, we round up our discussion and forward some testable propositions on talent-driven market structures for further empirical research.

2. The allocation of talent

A key determinant of the performance of an economy is the way in which talents are allocated. Baumol (1990) provides a historical overview of the allocation of entrepreneurial talents to different kinds of occupations and its consequences for economic development. Murphy et al. (1991) provide empirical evidence that economic growth is negatively affected in case talent is channeled towards redistributive activities instead of productivity-enhancing activities. In the current paper we concentrate on the question of who is going to be running firms and of what size and who is going to be employee and in which firms. Rosen (1982) characterized this question as an assignment problem. The solution of the assignment process provides a distribution of firm size and rewards. Furthermore, in case of multiple talents it provides a characterization of which of the talents are being used in the production process.

A solution to the assignment problem of individuals to be employer (self-employed) or employee in the case of one single 'talent' was provided by Lucas (1978) and Kihlstrom and Laffont (1979). Lucas derived the critical level of *managerial* ability above which individuals would be employers and below which individuals would be employees. The employees then had the same level of productivity irrespective of their level of managerial ability. Kihlstrom and Laffont instead concentrate on the degree of *risk aversion* and derived that the more risk averse individuals would become employees receiving a fixed wage while the less risk averse individuals would be running the firms. Rosen (1982) extended the individuals' talents to be multi-dimensional, one talent for each level of authority. In case these talents follow a one-factor structure he shows that individuals with highest general ability (factor score) are assigned to the top ranks of firms. The most talented persons are assigned to positions of greatest influence. Klepper (1996) introduces heterogeneous ability to *innovate* to explain which firms are likely to survive during industry evolutions. Gifford (1993) and Jovanovic (1994) have persons characterized by a combination of talents for specific tasks like management, innovation or production and derive the assignment of persons to become employee or employer.

In the current paper we assume that persons have multiple talents but that there is no one specific talent for ‘managing’.³ Individuals may choose to be self-employed and have a specific production technique combining their talents in an optimal manner. As an alternative they may choose to become an employee of such an employer and adjust to his or her production technique. In case the employer is more talented than the employee the latter may then achieve higher productivity than when being self-employed by benefiting from supervision. Both employer and employee profit from such a labor contract because the wage paid by the employer is in between the level productivity of the employee and his or her level of productivity at the second-best occupation (being either self-employed, working at another firm or an outside option).

The principle unit of analysis in our treatment is the set of (potentially) productive attributes (knowledge and skills) that each individual possesses. We call these attributes ‘talents’, and each individual is therefore represented by his or her ‘talent vector’ describing ‘how much’ of J possible talents an individual possesses⁴. Our contention is, that the allocation of these talents over firms and within firms is an important driving force behind changes in internal and external structures. Internally, the allocation of talent can take the form of hiring and firing or of people in higher hierarchical layers being replaced by people from lower layers. Externally, it can take the form of entry or exit or of take-over of one firm by another.

Each individual has three options: being an employee, being an employer or being self-employed without employees. The choice between being self-employed and working for an employer is made by comparing their own economic value in terms of productivity in each of the existing firms or by starting an own firm. When starting an own firm it may lead to employees of other firms or even other employers deeming it beneficial to join the newly started firm. In choosing between different prospective employers, the individual's talents determine their potential productivity for employers. Individuals and firms are attracted in

³ Various theories of multiple abilities or intelligences have been suggested in psychological research. An early example is Thurstone (1938) discriminating between seven primary mental abilities: verbal comprehension, verbal fluency, number, spatial visualization, inductive reasoning, memory and perceptual speed. Gardner's (1983) theory of multiple intelligences has seven distinct and independent kinds of intelligence: linguistic, logical-mathematical, spatial, musical, bodily-kinesthetic, interpersonal and intrapersonal. Guilford (1967) introduced a structure of intellect theory combining various aspects of intelligence for a total of 120 different abilities. Carroll (1993) has three dimensions of human intellectual competence: fluid intelligence (solving new and unusual problems), crystallized intelligence and visual-spatial reasoning. Sternberg's (1988) componential model also has three components, viz. processing, context and experiential components.

⁴ Note that our ‘talent’ concept is akin to that of the ‘resource’ defined at an individual level. In fact, our theoretical description can be conceived of as an individual level resource-based theory. However, the assumption of the non-imitable and non-transferable character of resources is relaxed in our conceptualization. After all: talents are transferred and traded, although they remain confined to individuals.

order to make the best of their talents. In their choice between firms, reward-maximizing individuals are predicted to be guided by two principles. *First*, individuals prefer to work for employers that possess comparable proportions of talents. There is an economic and a (related) social-psychological interpretation for this. From an economic viewpoint, individuals want to work where their specific mix of talents is best valued and the risk of carrying 'idle' talents is lowest. These conditions are best met with employers having comparable talent-proportions. Moreover, social-psychological research (Byrne et al., 1966) has demonstrated that people carrying comparable characteristics are more attracted to each other and communicate more effectively (Wagner et al., 1984; Zenger and Lawrence, 1989). This so-called similarity-attraction principle thus provides important (economic) gains in the *process* of cooperation (Smith et al., 1994). Note that the outcome of all this is a form of *social reproduction* (Kanter, 1977) or *self-cloning* (Hambrick et al., 1993) of the employer's talent proportions. This tendency is strengthened over time as employees learn, which brings us to the *second* choice-principle: individuals choose employers with the best possibilities to benefit from or learn and *develop* their talents, thereby increasing their productivity and economic value. With some employers there is more to gain in terms of productivity than with others. The intuition behind our model may be clarified using an example from academics. Traditionally, universities seek workers with talents in the area of research and/or teaching. Depending on the specific specialization of any university (employer) on either talent, individuals are attracted that have comparable talent-proportions. 'Research-academics' are therefore attracted to 'research-universities' and 'teaching-academics' to 'teaching-universities'. They fit best in those environments and can develop more easily along their specialized employers' talent vector.

To sum up the above, in our model the matching of individuals with employers is achieved by both parties optimizing their productivity. In doing so, they simultaneously optimize the current talent-fit and opportunities to develop their talents. Individuals consider their options for selling their talents, they choose to stay, change employer or start their own business.

From a dynamic perspective, flexibility of firms plays an important role in the model. Flexibility is a function of the talent distribution of the employees in relation to the vector of required talents as dictated by ensuing market conditions. The latter vector is taken to reflect the technological state of the world in which certain talents are more crucial to firm competition than others. In case employees are very specialized in certain tasks and fail to have a talent for other (or new) tasks, the firm structure is inflexible. This may not be a

problem during periods of stable demand and technologies where the ‘talent vector’ can simply be reproduced over time through selective hiring of quasi-identical people. The existent stock of talents is then exploited to the full. It may, however, be a problem during periods of industrial revolution (Piore and Sable, 1984) when the relative emphasis in required talents shifts and companies find themselves having redundant and/or lack of required talents. In the university-example: suppose that over time *management* talents are becoming increasingly important in addition to research and teaching talents. Universities are now at risk of not possessing these talents as they were never promoted⁵. We explore the impact of such technological shocks on the dynamics of talents within the firm and on the structure of the industry as a whole. We derive that the degree of flexibility at the organizational level has important consequences for survival and entry flows at the industrial level. We will now turn to the formal description of the model.

3. Model specification

3.1 The productivity of self-employed workers

Assume that each individual $i = 1, \dots, N$ has a J -dimensional talent vector $x_i = (x_{i1}, \dots, x_{iJ})$ being a realization of the random vector $X = (X_1, \dots, X_J)$. The elements X_j of the random vector are supposed to be identically and independently half-normally distributed.⁶ That is, they are distributed as the absolute value of a $N(0, \sigma^2)$ -variable. The mean of such a distribution is $\sqrt{2/\pi} \sigma$. The standard deviation is proportional to the standard deviation of the underlying normal distribution: $\sqrt{(\pi - 2)/\pi} \sigma$. The expected value of the squared value of a half-normally distributed talent vector element is: $E[X_j^2] = \sigma^2$.

We assume that at any moment in time only two elements of the vector are of importance in the production process. Without loss of generality, assume that these are x_{1i} and x_{2i} . Other

⁵ Note that those who accidentally *have* these previously ‘latent’ management talents now see a sudden rise in their potential productivity (and associated career opportunities).

⁶ It is likely that only those individuals will work in a specific industry that have talents at least higher than a certain cut-off, for example due to needing a certain level of education. In case a talent is normally distributed among individuals in general, the talent distribution of workers in an industry is therefore likely to be approximately half-normally distributed due to this cut-off point. Intelligence test scores often follow an approximately normal distribution, with expressions like “gifted” sometimes assigned to people with IQs over a certain point, such as 130.

elements of the talent vector are considered to be ‘latent’: they are individual resources, but not currently of importance in the production process. The productivity of an individual i who chooses to be self-employed depends upon the two elements of his or her talent vector and is taken to equal⁷: $x_{1i}^2 + x_{2i}^2$. This is the square of the length l_i of the line from the origin to x_i . It implies that self-employed individuals with different combinations of talent vector elements but equal talent vector length are equally productive. The expected productivity of a self-employed individual equals $E[X_1^2 + X_2^2] = E[X_1^2] + E[X_2^2] = 2\sigma^2$. The two-dimensional talent vector can also be described in terms of its length l_i and angle α_i with the horizontal axis. These are distributed as:

$$(1) \quad L = \sqrt{\sum_{j=1}^2 X_j^2} \sim \text{Rayleigh}(\sigma)$$

$$(2) \quad A = \arctan\left(\frac{X_2}{X_1}\right) \sim U[0, \pi/2]$$

The length and angle are related to the elements of the talent vector through $x_{1i} = l_i \cos(\alpha_i)$ and $x_{2i} = l_i \sin(\alpha_i)$. That is, the angle is a measure of the relative proportion of the two talent vector elements. The distribution function f of a Rayleigh random variable equals (see Johnson et al. pp.456-461):

$$(3) \quad f(x) = \frac{x}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right) \quad x \geq 0$$

The mode of a Rayleigh random variable is σ while the mean and standard deviation are $\sqrt{\pi/2}\sigma$ and $\sqrt{2-\pi/2}\sigma$, respectively. The p -th percentile of a Rayleigh-distribution equals:

$$(4) \quad F^{-1}(p) = \sigma \sqrt{-2 \ln(1-p)},$$

where F is the distribution function. It implies that the expected value of the order statistics (in descending order, i.e. $l_{[1]}$ being the highest value) can be approximated by:

⁷ Note that this specification implies a technological state of the world assigning an equal weight to either talent.

$$(5) \quad l_{[i]} = \sqrt{2\sigma} \sqrt{\ln(N) - \ln(i - 0.5)}.$$

for $i = 1, \dots, N$. Hence, the highest productivity in a sample of N individuals is approximately equal to $2\sigma^2 [\ln(N) - \ln(0.5)] = 2\sigma^2 [\ln(N) + \ln(2)]$.⁸ That is, the highest expected productivity increases with sample size N but decreasingly so.

3.2 The productivity of employed workers

Apart from being self-employed individuals may also be employed by another individual: the employer. In case individual s is employed by individual i his or her productivity depends on the extent of ‘adaptation’ and of ‘benefiting’. *Adaptation* means that individual s is to conform to the way in which individual employer i combines his or her talents to produce, i.e., to α_i . If the point (x_{1s}, x_{2s}) is above the line with corner α_i then the ‘productive’ talents are $(x_{1s}, x_{1s}x_{2i}/x_{1i})$, the rest of x_2 talents is ‘idle’. In case the point is below the line they are $(x_{2s}x_{1i}/x_{2i}, x_{2s})$, indicating ‘idle’ x_1 talents. That is, the further away the point from the employer’s productivity line l_i , the higher the loss in terms of unused, idle talents’ productivity. Productivity losses are lowest when employment is sought with employers that have relatively equal talent-combinations: this constitutes the concept of similarity attraction and social reproduction in our model. However, once employed by i , the individual s also gains from being in the firm by *benefiting* from being employed by individual i . It is assumed that certain organization efficiency-gains are available for the individual when he employs his talents in an organized context instead of on its own. This may take the form of reduced administrative burden and increased staff support, enabling them to get more out of their talents in an organized than in a stand-alone environment. Extensive human-resource policies and opportunities may further enhance what individuals make of their talents. The extent of benefiting depends upon the distance between the ‘productive’ talents of the employee and the talents of the employer. The productivity of the employee is enhanced by a fraction

⁸ The first ten order statistics of length are: 3.899 3.606 3.462 3.363 3.287 3.226 3.174 3.128 3.088 3.052. Gran (1992), formula (4.3.57), gives as an approximation of the expected value of the highest order statistic of the Rayleigh distribution the expression $\sqrt{2\sigma}(\sqrt{\ln(N)} + 0.2886/\sqrt{\ln(N)})$ which is very close to $l_{[1]}$ derived from equation (5).

κ ($0 \leq \kappa \leq 1$) of the distance, which is assumed to be equal across firms and workers. Hence the ‘productive’ talents (for above the line individuals) shift to:

$$(6) \quad (x_{1s}, x_{2s}) \rightarrow ((1-\kappa)x_{1s} + \kappa x_{1i}, (1-\kappa)x_{1s}x_{2i}/x_{1i} + \kappa x_{2i})$$

In case the benefiting fraction κ would be one, the employee would produce as much as the employer.

Thus, taken together, working at a firm has a positive and negative effect. The *positive* effect is that a person can enhance its productivity by ‘benefiting’ from being a member of the employer’s organization. This effect is dependent upon the size of the employers’ productivity l_i . The *negative* effect is that a person is restricted in the full use of his initial skills (adaptation). This restriction is a consequence of a firm employing the production technique that reflects *the employer’s* talents. This risk is minimized by choosing an employer that is most similar to the mix of talents of the individual (i.e., social reproduction). This similarity is reflected in the difference between employers and individuals’ α -values.

The production technology is assumed to be of a Leontieff type. Hence, the productivity of an employee is determined by the relatively least developed element of his/her talent vector. The productivity of individual s being employed by individual i can be derived to equal:

$$(7a) \quad \alpha_s \geq \alpha_i \quad \left(\frac{x_{2s}}{x_{1s}} \geq \frac{x_{2i}}{x_{1i}} \right): \quad l_i^2 \left[\kappa + (1-\kappa) \frac{x_{1s}}{x_{1i}} \right]^2 = l_s^2 \left[\kappa \frac{l_i}{l_s} + (1-\kappa) \frac{\cos(\alpha_s)}{\cos(\alpha_i)} \right]^2$$

$$(7b) \quad \alpha_s < \alpha_i \quad \left(\frac{x_{2s}}{x_{1s}} < \frac{x_{2i}}{x_{1i}} \right): \quad l_i^2 \left[\kappa + (1-\kappa) \frac{x_{2s}}{x_{2i}} \right]^2 = l_s^2 \left[\kappa \frac{l_i}{l_s} + (1-\kappa) \frac{\sin(\alpha_s)}{\sin(\alpha_i)} \right]^2$$

Equation (7) shows, as stated above, that the ‘employed productivity’ depends upon a) the ratio of the lengths of the talent vectors and b) the difference between the angles. It follows that when κ equals one (all employees take on the productivity of the employer), everybody has a higher productivity when having the most productive individual as employer, while when κ equals zero, there is no firm at which an individual can be more productive than by being self-employed. Hence, in equilibrium all are self-employed.

3.3 The choice between employment and self-employment: the one-employer case

In Figure 1 we give a graphical representation of the choice between employment and self-employment in particular. The axes show the amounts of talents x_1 and x_2 of individuals in a two-dimensional talent-space. Individual i 's (self-employed) productivity is determined by the length of the line (l_i) from the origin to their talent-point (x_{1i}, x_{2i}). We consider individuals with unit talent vector lengths (represented by the unit circle, indicating equal individual productivities) which have to choose between being employed by Z and being self-employed. Individual Z has a talent vector which is longer than that of individuals on the unit circle. When individuals choose to work for Z, they fully employ their most scarce talent relative to the employer: in case of individual A (with $\alpha_A > \alpha_Z$), this is x_1 , for individual B, (with $\alpha_B < \alpha_Z$) this is x_2 . The more individuals approach the axes, the more specialized they are in using either talent. As all individuals on the unit circle are equally productive, a talents' marginal productivity rises as it is more dominantly used. Individuals on the 45° line have equal marginal productivities for each talent.

We denote the individuals on the unit circle who are indifferent between being self-employed and employed by Z by A and B. Consider individual A with self-employed productivity l_a . If (s)he decides to work for Z, his/her productivity is determined by equation (7a). Graphically, (s)he migrates from point A to point Q (*adapting*) and then benefits some fraction κ of the distance QZ along Z's talent vector. As a consequence, A's productivity as employee is equal to that of an individual P on the unit circle being self-employed (in formula: $l_q[\kappa(l_z/l_q) + 1 - \kappa] = l_q + \kappa[l_z - l_q] = l_p$). Hence, A is indifferent between self-employment and employment. All individuals on the unit circle between A and B will prefer to work for Z over self-employment. Note that the length of PB is much smaller than that of PA, indicating that Z as a relatively x_1 -intensive producer (or 'specialist') has more difficulty in attracting even more strongly specialized people (below z) than it has in attracting less specialized (or 'generalist') people (above z). Or, stated the other way around, more x_1 -specialized people are less likely to work for the specialist Z than are less x_1 -specialized people. The reason is, that for x_1 -specialized people adapting is no longer balanced by benefiting. For very intensive x_1 -specialized producers (like B'), productivity suffers stronger from 'handing in' their specialized than their non-specialized talents simply because they rely on them so much and the marginal productivity of their specialized talent is higher than that

of their non-specialized talent. They are therefore better off using their specialized talent to the full and be self-employed.

In the one employer case we may derive some analytical results concerning the expected market share of the employer in terms of employment. We will focus upon the question of the size of the expected market share of the person with the longest talent vector ($l_{[1]}$) being the one employer. Consider the probability that an individual s is more productive being an employee. It equals:

$$(8) P[\alpha_s \geq \alpha_{[1]}] P[\alpha_s < \arccos\left(\frac{1 - \kappa l_{[1]}/l_s}{1 - \kappa} \cos(\alpha_{[1]})\right) | \alpha_s \geq \alpha_{[1]}] + \\ P[\alpha_s < \alpha_{[1]}] P[\alpha_s > \arcsin\left(\frac{1 - \kappa l_{[1]}/l_s}{1 - \kappa} \sin(\alpha_{[1]})\right) | \alpha_s < \alpha_{[1]}].$$

Two interesting cases are when the most productive individual either is on the horizontal or vertical axis ($\alpha_{[1]} = 0 \vee \alpha_{[1]} = \pi/2$) or has equal talents in both dimensions ($\alpha_{[1]} = \pi/4$).

In the first case the probability reduces for both axis to $\frac{2}{\pi} \arccos\left(\frac{1 - \kappa l_{[1]}/l_s}{1 - \kappa}\right)$. In the second case it reduces to $\frac{4}{\pi} \arccos\left(\frac{1}{\sqrt{2}} \frac{1 - \kappa l_{[1]}/l_s}{1 - \kappa}\right) - 1$. The probability is less in the latter case of

the longest talent vector being diversified in its elements.

A numerical example shows how the probabilities depend upon the benefiting parameter κ . Assume that there are one thousand individuals ($N = 1,000$) and that we want to know the probabilities of the 2nd, 10th and 100th talented in order to be more productive when being an employee of the person with the longest talent vector for both the axis and 45 degrees case. We have that $l_{[1]}/l_{[2]} = 1.081$, $l_{[1]}/l_{[10]} = 1.278$ and $l_{[1]}/l_{[100]} = 1.815$ from equation (5). We consider values of κ equal to 0.1, 0.4 and 0.7. The probabilities are given in Table 1. We find that increases in the value of κ lead to increased probabilities of becoming an employee. At a value of 0.7 the 100th talented in order has probability one and, hence, the expected market share of the one employer is at least 90%.

Table 1: Probabilities of employment in case of one employer

κ	Axis firm ($\alpha_{[1]} = 0 \vee \alpha_{[1]} = \pi / 2$)			Equal talents firm ($\alpha_{[1]} = \pi / 4$)		
	[2]	[10]	[100]	[2]	[10]	[100]
0.1	0.085	0.159	0.273	0.011	0.039	0.111
0.4	0.210	0.394	0.698	0.067	0.218	0.581
0.7	0.398	0.771	1	0.222	0.680	1

We may also compute the probability that each of the individuals is more productive when being an employee of the 1st ordered, or the probability that a firm will turn out to be a

natural monopolist. This probability for the axis firm equals $\prod_{\substack{r=2 \\ l_{[r]} > \kappa l_{[1]}}} \frac{2}{\pi} \arccos\left(\frac{1 - \kappa l_{[1]} / l_{[r]}}{1 - \kappa}\right)$.

Assume again that N is 1,000. The values of $l_{[1]}$ through $l_{[5]}$ are then 3.899, 3.606, 3.462, 3.363 and 3.287 from equation (5). In case of a parameter κ equal to 0.85 we have that only values of r smaller than five have to be considered. The probability equals 0.49. Therefore, there is a chance of one in two that all are more productive when being employed by the most productive.

We now turn to the case of two firms. This provides an intuitive background for the equilibrium configuration as discussed in Section 4.

3.4 The choice between employment and self-employment: the two-employer case

Take as a next step the two-employer case. Individuals choose between being self-employed or being employed by one of two firms. As shown above, in general, individuals are attracted to an employer because of two factors: the similarity of the proportion of talents, i.e. the angle, and the length of the employer's talent vector (benefiting possibilities). Roughly speaking, if we consider the employers with longest talent vectors, they will divide the group of individuals who become employees in the way of a pie chart. This notion is supported by considering the individuals with angles *in between* those of firms i and j . Individual s is indifferent between employer i and j when $l_i [\kappa + (1 - \kappa)x_{1s} / x_{1i}] = l_j [\kappa + (1 - \kappa)x_{2s} / x_{2j}]$.

This gives the following linear relationship:

$$(9) \quad x_{2s} = \frac{\kappa(l_i - l_j)x_{2j}}{(1 - \kappa)l_j} + \frac{l_i x_{2j}}{l_j x_{1i}} x_{1s}$$

This line of indifference starts in the origin only in case the two employers have equal talent vector length ($l_i = l_j$), so in general there are no ‘perfect’ pies. In fact, there are six relations between x_{1s} and x_{2s} that may matter when having two employers and the possibility of self-employment. Two for the discrimination between self-employment and the first employer, similarly two for the second employer and two for the discrimination between the two employers. We may need two relations (i.e., one additional to (9) above) to discriminate between the employers because there may be individuals choosing the employer with an angle *further* away from his or her own than the other, because the former has a much higher talent vector length. This is an important point, as these individuals accept the adaptation loss of having a very *dissimilar* employer for the benefiting gain of a highly *talented* employer. In case $\alpha_j > \alpha_i$ and $l_j < l_i$ individuals with an angle larger than that of firm j are indifferent in case $l_i[\kappa + (1 - \kappa)x_{1s} / x_{1i}] = l_j[\kappa + (1 - \kappa)x_{1s} / x_{1j}]$, or the following horizontal line:

$$(10a) \quad x_{1s} = \frac{\kappa(l_i - l_j)}{(1 - \kappa)(l_j / x_{1j} - l_i / x_{1i})}$$

In case $l_j > l_i$ we get a vertical line instead:

$$(10b) \quad x_{2s} = \frac{\kappa(l_j - l_i)}{(1 - \kappa)(l_i / x_{2i} - l_j / x_{2j})}$$

The indifference curve discriminating between self-employment and employment at firm i consists of two parts: one for individuals with an angle larger than the employer and one for those with an angle smaller. For the first part we have the curve:

$l_s^2 = l_i^2[\kappa + (1 - \kappa)x_{1s} / x_{1i}]^2$, or:

$$(11a) \quad x_{2s} = \sqrt{l_i^2(\kappa + (1 - \kappa)x_{1s} / x_{1i})^2 - x_{1s}^2} \quad 0 \leq x_{1s} \leq x_{1i}$$

For the second part we have the curve $l_s^2 = l_i^2 [\kappa + (1 - \kappa)x_{2s} / x_{2i}]^2$, or:

$$(11b) \quad x_{1s} = \sqrt{l_i^2 (\kappa + (1 - \kappa)x_{2s} / x_{2i})^2 - x_{2s}^2} \quad 0 \leq x_{2s} \leq x_{2i}$$

Similar expressions can be derived for firm j , leading to indifference curves (11a') and (11b').

We illustrate all this graphically in Figure 2. Consider the case of two employers, Y with talent vector (1,2) and Z with talent vector (3,1). We will first discuss the two lines discriminating the choice for employer Y or for employer Z . After that we will discuss the curves discriminating between self-employment and employment. The first line of discrimination between Y and Z for individuals is made by curve (9). As the latter firm has a longer talent vector and a smaller angle, we use equation (10a) instead of (10b), leading to an additional line of discrimination (10a). There are four indifference curves between employment and self-employment, viz. (11a) through (11b'). Figure 2 shows the areas of self-employment, choice for employer Y and choice for employer Z . The figure shows three areas in which one may expect self-employed given that Y and Z are the two longest talent vector individuals. Individuals which are very talented in the first dimension but much less so in the second, vice versa, and those that have talents in both dimensions and have a talent vector length just short of that of that of the employer(s). The first two groups are *specialists* in one dimension, the second group are *generalists*. As the total area enclosed by employer Z is much bigger than that of employer Y , Z has a higher 'market share' in terms of employed individuals than Y .

It is clear that for both specialists and generalists it can be profitable to work for an employer. However, for the more talented specialists it is relatively more attractive to be self-employed than for talented generalists. In other words: generalists need to be more talented in order to be self-employed than generalists. This is so, because generalists have relatively much to gain (benefiting) and less to lose (adapting) by being employed compared to specialists. This means that organizations tend to be populated more by generalist people whereas self-employed tend to be more specialist. Inversely stated: specialists are more likely to be self-employed than generalists. As a consequence, those generalists that *are* self-employed need to be much more talented, in the sense of having a longer talent vector, than self-employed specialists.

As shown, employer Z 's market share is much higher than that of employer Y . Note that even individuals in an area close to the vertical axis prefer to work for Z instead of for Y . The benefit of lower adaptation when working for the more similar employer Y does not compensate for the higher possibilities of benefiting from the more talented Z . Thus for weakly talented⁹ individuals it may pay to disregard the compatibility of a prospective employer as long as the latter is talented enough to offer benefits that compensate for the poor match. The (less talented) employer Y therefore mainly attracts a lower variety but a higher average level of talents than the more talented Z . This is so, because in choosing to work for less talented employers, similarity is the dominant choice-consideration. Social reproduction is thus weaker, on average, in case of highly talented employers. The latter attract much more 'opportunists' in the form of more diverse and low average talented employees.

4. Equilibrium allocation of talents

We assume that each individual seeks that position that will make him or her as productive as possible. This may mean that the person becomes self-employment and leads a firm, possibly with employees. The person then gets a reward equal to his or her productivity. The alternative is that the person joins a firm. The reward is then equal to the wage that the employer pays. The exact wage level is not determined in the model but will be in between the level of productivity of the employee when working at the firm and the second-best alternative for the employee. This labor contract is beneficial both for the employer, who receives the difference between the wage and productivity level and for the employee, who receives a wage level which is higher than his or her level of productivity at any other firm or when being self-employed. The precise wage level is determined by negotiation between employer and employee.

The equilibrium configuration given the talent vectors of N persons can be derived in several ways, but a very straightforward one is the following. The person with the longest talent vector (l_{11}) will always be self-employed because he or she can never gain by being an employee. This is the first firm. The person with the second-longest talent vector (l_{12}) may benefit from joining the firm or will start a second firm, the latter being most likely. Then we turn to the other persons, in order of the length of their talent vectors. Each time when a

⁹ Note that we speak in strict static terms, here. From a dynamic perspective, these are better labeled weakly *developed* talents.

persons is more productive when starting a firm compared to being employed by one of the already existing firms, a new firm is added. In this section we show that this procedure of deriving the equilibrium gives an optimal configuration in the sense of everybody having the highest productivity possible.

Theorem 1: An individual's choice for employment instead of self-employment may only enhance his or her productivity when the employer is more productive in case both individuals would be self-employed.

Proof: The value of κ for which the productivity would be maximal is unity. For both cases

$$\frac{x_{2s}}{x_{1s}} \geq \frac{x_{2i}}{x_{1i}} \quad \text{and} \quad \frac{x_{2s}}{x_{1s}} < \frac{x_{2i}}{x_{1i}} \quad \text{this maximal productivity of individual } s \text{ when employed by}$$

individual i with a smaller length of the talent vector is l_i^2 which does not exceed l_s^2 . Q.E.D.

Corollary 1: The individual with the largest length of the talent vector can never be as productive than when being self-employed.

Theorem 2: In case an individual is most productive when employed by some individual, then the latter individual is most productive being self-employed.

Proof: consider the case of three individuals with $x_{11} < x_{21} < x_{31}$ $l_1 < l_2 < l_3$. We are now

going to proof that if individual 1 is more productive with individual 2, then individual 2 is not more productive with individual 3. We have that

$l_2[\kappa + (1 - \kappa)x_{11}/x_{21}] > l_3[\kappa + (1 - \kappa)x_{11}/x_{31}]$. Now reset $x_{11} = sx_{21}$ $s < 1$, then the

inequality implies that $l_2 > l_3 \left[\frac{\kappa + (1 - \kappa)sx_{21}/x_{31}}{\kappa + (1 - \kappa)s} \right]$. This is true when

$\kappa(1 - s)(1 - x_{21}/x_{31}) > 0$. This is always the case. Q.E.D.

In the first theorem we show that persons will never be employees of an employer with a shorter talent vector. This confirms the choice of starting with the longest talent vector and proceeding with ever shorter talent vectors. Persons who have become self-employed will remain to have that position when persons with shorter talent vectors are considered. In the

second theorem we show that at the end when it has become clear which persons are employers, that all of the other persons will find it profitable to join one of these firms.

Consequence: The equilibrium is *optimal* in the sense of every worker achieving his or her maximum reward and total productivity being as high as possible.

5. Comparative static analyses

In the current section we discuss some characteristics of the equilibrium market structure given the individuals' talents. We discuss the dependency of market shares of firms on the length of the employers' talent vectors. Next, we show (typical) results of a single simulation to achieve insight in the (typical) equilibrium configuration. Finally, we provide averages of a set of one thousand simulations to see the ranges in simulated market structures.

5.1 Talent and market share

In Figures 3 and 4 we show what happens to the market share distribution in case individual Y is replaced by a more talented individual, either Y' at (2,2) or Y'' at (3,2). If we consider Figures 2 and 4 we see that the market share in terms of employees rises disproportionately with the length of the talent vector. That is, *firm size is much more skewed to the right than the talent distribution*. Once the employer surpasses individual Z in terms of productivity, as in Figure 4 it starts employing 'opportunists' from Z and the reverse qualitative situation of Figure 2 results. If this were a truly dynamic process (for instance a new employer Y'' enters) this would amount to migration (turnover) of predominantly low talented, dissimilar (to Z) individuals from Z to Y . Thus, migrating individuals are more likely to be of the more dissimilar (to Z) and low talented type. Note that this is simply the reverse of the attraction that leads to social reproduction: the most similar are most attached and are less likely to leave.

5.2 A typical simulated market structure

In this section we discuss results of a typical simulation. We have drawn a single sample of one thousand individuals (i.e., talent vectors) from the two-dimensional half-normal distribution (with $\sigma = 1$). The value of κ is assumed to be one third. We find the following results. The total number of firms in equilibrium is thirteen, only four of which have a market

share in terms of employment that exceeds 10%. Figure 5 shows the locations of the thirteen firms, two of which have no employees at all. As expected from Figures 2 through 4 we find some very specialized small firms near the axes and some generalists between the large firms. The talent vectors of the large firms roughly lie on a circle and, therefore, the lengths of the talent vectors are not too different. Hence, we expect pie-like distribution of the employees between the employers. This is confirmed by Figure 6 in which we show the employees of two of the large firms. The employees of the firms are located near the line connecting the origin and the talent vector point. However, the employees are not symmetrically distributed on both sides of this line. The majority of employees is located to the side of the line closest to the 45 degrees line. Both specialized firms have more difficulty attracting *more* specialized than *less* specialized people.

The model gives rise to a market structure that is characterized by a co-existence of a few small generalists, some big moderately specialized firms plus a number of very specialized companies.¹⁰ The firms employ a variety of employees with one common characteristic: they have similar values of the proportion of talents, or α_i . A worker with a talent vector of (2,1) is predicted to be an employee of an employer with talent vector (4,2) if the alternative is an employer with talent vector (2,4). The rationale behind this from the viewpoint of social reproduction can be explained by considering the firm to be hierarchies with layers characterized by different amount of talents (see also Rosen (1982)). Assume the span of control of layer k to be S_k . The top layer is the employer. The second layer consists of the S_1 most talented employees in the organization. Their talent vector (in as far as it concerns the productive talents) resembles that of the employer very much. The third layer consists of the $S_1 S_2$ next most talented employees. They are very close to at least one of the employees in the second layer of the hierarchy, and so on. Therefore, the least talented person in the hierarchy may have little in common with the employer in a *direct* sense, but as each layer is ‘socially reproduced’ the two will find themselves in the same organization as their *proportion* of talents is similar.

In Figure 7 we show the evolution of the equilibrium number of firms when individuals are added to (i.e., enter) the industry one by one. The general pattern is that of strong increase of the number of firms in the beginning and slow increase later on. Self-employment is an increasingly unattractive option as the probability of a person being more

¹⁰ Such a coexistence of specialists and generalists is also predicted by the ‘resource partitioning’ theory (Swaminathan (1995)). See also Bonaccorsi and Giuri (2000).

talented than the already incumbent employers goes down. Because we assume that the talent vectors do not change, i.e. there is no learning, there is no advantage of entering early or late. The number of firms fluctuates as individuals enter because there are both opportunities to become an employer (i.e. to start a firm) *and* to switch from being an employer to being an employee (i.e. to ‘give up’ a firm). Thus firms form and disband (or merge), shrink and expand continuously as individuals seek the best way of employing their given talents.

Table 2: Entry and exit of employers in a typical simulation

Individuals	Firms	Entry	Exit
50	6	9	3
100	5	5	6
150	11	6	0
200	10	4	5
300	11	3	2
400	9	5	7
500	9	2	2
600	10	3	2
700	11	1	0
800	12	3	2
900	14	2	0
1000	13	4	5

We show the patterns of entry and exit of this typical simulation exercise in Table 2. The pattern shows that there is first a period of high net entry rates (as a percentage of the number of workers in the industry) up till about the 150th worker, though that during this period there is also considerable selection of firms going on. After this there is a period of high entry and high exit rates, up till about the 400th worker. After this the entry and exit rates become much smaller. It is clear from the table that entry and exit rates in periods are positively correlated, especially after the first period of initial high net entry. The positive correlation between these rates is typically found in industrial analysis (see e.g. Dunne et al. (1988)).

5.3 The range of simulated market structures

The disadvantage of a single simulation is that it may reflect idiosyncrasies. Therefore, we ran one thousand simulations to find the general patterns in key descriptives of the industry. We focus on the number of firms, the extent of concentration and on entry and exit flows. The rising pattern of firm numbers as a function of the number of persons (N) is confirmed. The

results of these simulations are depicted in Figure 8. It shows the average number of firms in equilibrium to rise from 7.2 for 50 individuals to 15.6 for 1,000 individuals. The standard deviation also increases, from 2.0 for 50 individuals to 3.3 for 1,000 individuals. The extent of concentration decreases as the number of individuals goes up. A reason for this is that the difference between the highest vector length, the second-highest vector length, etc. decreases as N increases. Figure 9 shows how the average Herfindahl Index decreases from 0.346 for 50 individuals to 0.272 for 1,000 individuals. In Figure 10 we present the rates of entry and exit for the first thirty individuals who enter an industry. We find the net entry rate (entry minus exit) to decrease over time, on average. This is a consequence of the average entry rate to drop strongly from about three quarters for the second individual to enter the industry to about 0.2 for the 30th individual. The average exit rate also declines, but much less strongly.

An increase in the number of individuals also leads to an increase in the average productivity. In case everybody would be self-employed we expect this to be two (because σ is one). In case of 50 employees we find it to be 2.92 on average. This increases to 3.14 for 100 employees, 3.74 for 500 employees and 3.98 for 1,000 employees. There are two reasons for this increase. First, more individuals means higher extreme values (see equation (5)). Second, more individuals means that more angles will have individuals with high talents. An industry in which many people work will be more productive on average than a similar industry in which few people work, *ceteris paribus*, because (i) the employers are, on average, more talented because there are more persons to select them from and (ii) there will be more firms in equilibrium for the employees to choose the most adequate from.

Table 3: Comparative static analysis: number of firms and Herfindahl index

N	$\kappa = .10$		$\kappa = .30$		$\kappa = .50$	
	#	H	#	H	#	H
50	9	0.1824	6	0.3200	4	0.5496
100	14	0.1590	8	0.2970	5	0.5582
200	18	0.1563	10	0.4000	5	0.7597
300	22	0.1308	12	0.2886	8	0.5391
400	25	0.1230	13	0.3008	8	0.5546
500	26	0.1207	13	0.2928	9	0.5420

Finally, we consider changing the value of κ . Remember that κ represents the amount of benefiting individuals experience from working for an employer. The effect of an increase in this parameter appears to be quite straightforward as we consider the two extreme

cases. A κ of zero makes everyone self-employed, a κ of one makes everyone working for the individual with the highest length of talent vector. For a simulation with other values see Table 3. It appears that the Herfindahl index is not too far from the value of κ when the number of persons N is sufficiently large. These findings are interesting as they show that the number of firms in the market decreases as the general possibilities of developing talents within firms increase.¹¹ As a consequence, industries characterized by intensive human resource programs are expected to consist of fewer firms and higher concentration.

6. Dynamic analyses

There are various ways of introducing dynamics into the model. We will discuss four factors: two forms of learning (extending capabilities), specific and not specific to the firm, creative destruction and retirement. In the first case, talents are permanently developed through learning beyond mere benefiting. In the second case, the firms (employers) progress in terms of capabilities as they are active in applying them ('learning by doing'). In the third case the relative productivity of talents shifts and a thus far latent talent becomes productive. In the last case we assume that workers retire after a certain period of time and exit the industry. Finally, we will discuss reasons for disequilibrium and ways that equilibrium can be restored.

6.1 Learning by doing or firm-specific learning

Up till now we assumed that the talent vectors of individuals were constant over time. In this section we discuss how the talent vector of *employees* and *employers* may change through *learning*. In section 3.2 we discussed how the productivity of individual s benefited from being employed by an individual i . The talent vector of employee s was assumed not to change as a result of benefiting from the employer's talents. This assumption excludes any learning-by-doing or learning-by-example. It is safe to assume that in reality employees learn, either consciously or unconsciously, and this learning can easily be examined in the context of our model.

Assume as a first variant that employees benefit by a fraction κ of the difference between their productive talents and the talents of the employer. When leaving the company, the employee was assumed to return to his or her original talent vector. However, if there is

¹¹ Note that we assumed equal κ across firms

learning employees may permanently extend their capabilities whilst being employed and thus be better-off standing alone after employment compared to their talents upon entering. One variant of such a process is depicted in Figure 11. If employee Q has talent vector point 1, (s)he increases the first talent vector element by working in the firm run by employer Z who has a much higher level of the first talent vector element. The second element of the talent vector is not changed, however, because much of it is left 'idle' in the production process. Through learning the talent vector of Q shifts to point 2 after one period. In the second period a similar process takes place, though the extent of learning is less. This process takes place until the talent vector will have similar talent proportions as that of the employer Z (i.e. $\alpha_Q = \alpha_Z$). From that moment on, employee Q will learn in terms of both talent vector elements. This learning process leads to persons becoming more and more productive in the organization in which they work. Note that the parameter κ now not only indicates the extent of benefiting but also the rate of learning. That is, employees benefit in two ways: instantaneously by having increased productivity and sequentially by having increased talent(s). The employee Q's stand-alone option (1...4) against which employment at either employer is rated, gets closer and closer to the current employer. This means that the attractiveness of other employers starting in the area around 1, 2 and 3 *decreases* in comparison to the static model. Thus, through (firm-specific) learning employees have lower incentives to switch employers than in case of mere benefiting. More possibilities for firm-specific learning thus leads to lower inter-firm mobility.

A disadvantage of the variant described above is that the rate of learning is equal to the extent of benefiting. In addition, it is hard to find a comparable learning rule for employers as they have no-one to benefit from. Therefore, we will use a variant of firm-specific learning in which a new parameter is introduced: the extent of learning parameter λ . The learning process for employees in the second variant is shown in Figure 12. In case the employee s has a ratio of x_{2st-1} / x_{1st-1} in period $t-1$ exceeding that of his employer i then:

$$(12a) \quad x_{1st} = (1 + \lambda \sin(\alpha_i))x_{1st-1} \quad \wedge \quad x_{2st} = \text{MAX} [x_{2st-1}, (1 + \lambda \cos(\alpha_i))x_{1st-1}x_{2it-1} / x_{1it-1}]$$

This case is shown in Figure 12 as the individual showing an increase in the first talent vector element only. In case the employee has a ratio of x_{2st-1} / x_{1st-1} less than that of his employer then:

$$(12b) \ x_{2st} = (1 + \lambda \cos(\alpha_i))x_{2st-1} \quad \wedge \quad x_{1st} = \text{MAX} [x_{1st-1}, (1 + \lambda \sin(\alpha_i))x_{2st-1}x_{1it-1} / x_{2it-1}]$$

This case is shown in the figure as the employee with an increase in the second vector element only. Employees that differ from their employer in the proportion of talents are likely to learn in one dimension only, that is the critical talent dimension, because of either courses offered in the firm exactly to improve this talent dimension or through learning-by-doing, as the employee gains knowledge in the critical talent dimension. In case the employee's talent proportion is close to that of his employer, then he may actually learn in both dimensions. In case an employee has exactly the same talent proportion ($\alpha_s = \alpha_i$) then we have:

$$(12c) \ x_{1st} = (1 + \lambda \sin(\alpha_i))x_{1st-1} \quad \wedge \quad x_{2st} = (1 + \lambda \cos(\alpha_i))x_{2st-1}$$

The consequence of equation (12) is that employees who have the exact same proportion of talents learn more than other employees in a firm. The reason is that both talent vector elements are used completely providing more opportunities for learning-by-doing.

6.2 Learning not specific to the firm and learning by the employer

If learning was restricted to employees, it is clear that the 'static' employer would have a strong disadvantage. However, firms (i.e., employers) themselves may also develop through learning-by-doing (or learning-by-managing), in the sense that the employers' talent vectors may become multiplied by a certain (fixed) value (λ) each period. The employer may, in that case, 'stay ahead' of his employees. Graphically (in the first variant), we would see Z's talent vector be enlarged in each period, progressing along an imaginary line from the origin through the original talent vector position of employer Z. Note that this type of technological progress when being an employer immediately implies a first-mover advantage for the first employers to emerge.¹²

The way in which the employer learns is in fact independent of his firm: his talent vector just becomes larger. By allowing employees also to have this kind of learning, some

¹² Such a first-mover advantage is also important in the Klepper (1996) model of industry evolution. In the Klepper model the rationale behind this advantage is economies of scale in R&D. In service industries firms may achieve first mover advantages through standardization and economies of scale in marketing and distribution. Szymanski et al. (1995) perform a meta-analysis and find that, on average, earlier entry is associated with greater market share. They discuss marketing mix elements which may account for this finding.

consequences of different learning processes can be investigated. For the case of employer i , we have learning equation (13):

$$(13) \quad x_{1it} = (1 + \lambda \sin(\alpha_i))x_{1it-1} \quad \wedge \quad x_{2it} = (1 + \lambda \cos(\alpha_i))x_{2it-1}$$

It is logical that this equation resembles that of equation (12c) corresponding to an employee having the same proportion of talents as the employer because in fact the employer may be seen as his own employee performing tasks for the company.

We have performed a simulation exercise for 20 periods and a learning parameter λ of 0.02 and found that the mean number of firms (over 100 simulations) decreased over time from 10.74 in the first period to 10.35 in the 10th period and 9.72 in the 20th period. In contrast, if every person would learn according to learning rule (13), then the average number of firms increases from 10.74 in the first period to 11.34 in the 10th period and 11.33 in the 20th period. Hence, firm-specific learning leads to less firms in the market while *general* learning (firm-unspecific) leads to an increase in the number of firms.

We have also considered the consequence of an increase in the learning parameter. In the case of employees having a learning rule according to equation (12) and having a learning parameter of 0.04 instead of 0.02, we have the average number of firms decreasing from 10.74 in the first period to 9.79 in the 10th period and 8.08 in the 20th period. In addition, the Herfindahl index also becomes much higher in the 0.04 learning parameter case when compared to the 0.02 learning parameter case. When the learning parameter is high, technological progress should also be fast to prevent talented employees from spinning off. The result indicates that in industries where technological progress is key, first mover advantages are important. This is in conformity with work by Klepper (1997) and Bonaccorsi and Giuri (2000).

6.3 Creative destruction

Another way of introducing dynamics is to have *creative destruction*. In our model this would be represented by changes in the relative importance within the talent vector instead of changes in the talent vectors themselves as was the case in the previous two sections. To take an extreme case, suppose the first knowledge element x_1 stops to be productive and is replaced as being productive by another element of the talent vector, e.g. x_3 . We assume that

the second talent is unaffected by the creative destruction. The shift in the importance of the talent vector elements will clearly affect the market structure as the production function in terms of talents is rewritten. Individuals possessing the formerly latent talent in sufficient amount will experience a rise in productivity, whereas others that strongly relied on the now obsolete talent vector element will loose. Note that those individuals with high values of x_3 (and low values of x_1) are likely to profit from the shift. Individuals that became employer due to their high levels of x_1 stand a great risk of losing their employer-status unless they, ‘accidentally’, also have a high value of x_3 . An employer using a production technology in which extensive use is made of element x_1 is likely to loose ground.¹³ The individual may become an employee of an individual who earlier on was his own employee, reversing the original situation.¹⁴

In our example, the change in the industry structure is somewhat moderated by the fact that the second talent vector element remains important. Thus, at least they tend to keep clustering around that talent¹⁵, forming a range of comparable organizations (i.e., talent/people clusters) as in the former situation. In technical terms: the correlation coefficient between talent vector lengths in the old and new situation (effectively measuring the extent of destruction) amounts to 0.5. This is why many firms do not break down completely. Moreover, firm size plays an important role: large firms also have large pools of talented people. In equilibrium the firm with a very talented employer has many employees and it is likely that one of those will turn out to be an employer ‘of the future’, safeguarding the company’s future success. The probability of survival of large firms is larger than for small firms as they have more access to talent. This leads to an important conclusion. Firms that strive for diversity may perform worse on the short term but may have a better performance on the long term (not too much concentration on core activities). This confirms the main

¹³ Such a firm may suffer from a ‘competency trap’: firms have specialized over time to such an extent that they fail to adjust to radically new environments. Firms have been found to have difficulty to change in fundamental ways from the experience with which they entered the industry (Klepper and Simons (2000b)).

¹⁴ It is important at this point to identify a subtle issue in the model that is of a more empirical and philosophical nature. Our model has identified a ‘firm’ by the ‘employer’ in charge. As long as the employer stays in office (as in the basic model version) this is not problematic. However, we run into a conceptual problem when the employer leaves or is ‘replaced’: are we then still talking about the same firm? This problem is of a broader methodological nature: can we speak of the same organization at two points in time when a high percentage of people have left and are replaced by others? We tend to do so in practice when we refer to companies like Ford and (Dutch) Philips, named after the founders, treating them as comparable entities now and decades ago, whereas they have profoundly different qualities. In the end we usually identify firms or organizations by their names.

¹⁵ Whether or not this moderation occurs simply depends on the scale or depth of creative destruction. When all talent-weights are altered significantly, we are left with a complete reshuffling of people and industry character.

findings of a large body of research in economics and organization science, showing and explaining a temporal trade-off between performance and adaptation (Van Witteloostuijn, 1998).

6.4 Flexibility and retirement

An obvious origin of dynamics is retirement of workers. Especially when employers with many employees retire, the industry equilibrium configuration may alter drastically. The consequences of retirement are also related to the introduction of learning or that of creative destruction. In case of a period of creative destruction we expect employers to be replaced (retire) earlier.¹⁶ When there is *no learning*, it can be hard for the employer when he or she wants to retire to find an adequate replacement, because the second-best worker in the company may be much less talented. In Figure 6 we find the example of the firm with 20% share of employment, but with the talents of the employer far in excess of even his or her employees with the highest talent vector elements. The other firm in the figure, with 15% employment share, has less distance between the talent vectors of employer and most talented employees. Retirement may affect the equilibrium configuration so that spin-offs of workers or even a take-over are needed to restore equilibrium. In case of *learning* the second-best worker will be much alike, in terms of the productive talent vector elements, as the employer, so we do not expect to many changes.

6.5 Disequilibrium and ways of error correction

Given the talent vectors of the individuals and market settings (demand, technological trajectory, regulations, etc.), the equilibrium configuration of employers and employees is unique and optimal. However, there are various reasons why the equilibrium is not attained or why the equilibrium may change over time. The equilibrium may not be attained because there are costs involved in inter-firm mobility or information gathering or because individuals differ in the degree of risk aversion.¹⁷ Equilibrium may change over time due to creative

¹⁶ A key reason for early retirement in periods of creative destruction is that ‘captains of industry’ have a lot of experience (crystallized intelligence), but may be less able to solve new and unusual problems (fluid intelligence). Horn (1985) and Horn and Noll (1994) find that fluid intelligence decreases from early adulthood on while crystallized intelligence has a tendency to increase.

¹⁷ Kihlstrom and Laffont (1979) argue that the degree of risk aversion is the key determinant of who becomes employer (the least risk averse workers) and who becomes employee.

destruction, retirement, government regulation (anti trust), entry of new workers into the industry, (unjustified) take-over activity, etc.

There are two distinct ways in which the industry can adjust towards the new equilibrium configuration, viz. through *intra-organizational mobility* and through *inter-organizational mobility*. It is likely that in case the new and old equilibrium configurations are very different, both ways of adjustment are needed. In case intra-organizational mobility is low, for example due to inflexibility in individuals moving from one layer of the hierarchy to another layer (and replacing the CEO), this will lead to opportunities for new firms, possibly spinning-off from the original company (inter-organizational mobility). Therefore, the extent of new entry during periods of radical technological changes is determined strongly by the extent to which incumbent firms are flexible enough to adjust to the new environment. An important element of inter-organizational mobility are take-overs. In case an employer retires and the new employer is less talented, the firm may be taken over by a ‘neighboring’ firm. Part of the firm which has been taken over may then consists of employees quite far away from the new employer in terms of their talent proportions and either start a new firm themselves (e.g. MBO) or are incorporated in another ‘neighboring’ firm (at the opposite side of the first ‘neighboring’ firm). Take-overs are therefore likely to be followed by new take-overs or spin-offs as the composition of employees in terms of talents has to be reconfigured to attain the new equilibrium.

7. Conclusions and propositions for further research.

Modeling industry dynamics from an individual viewpoint appears to be a fruitful and inspiring endeavor. As individuals and organizations seek a match between talents and associated productivity typical organization and industry level outcomes can be predicted. As we tried to make clear from the onset, talent-based competition will be more descriptive of some industries than others. We expect that it will be most useful and recognizable in professional industries, sometimes called 'people-industries'. Examples include innovative technical and service industries where competition for specifically talented individuals is fierce, e.g. IT-professionals, architects, lawyers and medical practitioners. In fact, our model can be viewed as sharing important elements of the Medieval ‘guild’ system: individuals start as an employee to learn from the ‘master’ (employer) and the best of those start firms on their own, sharing many elements of the firm which the individuals came from.

Below we list a set of testable propositions that our model gives rise to. Some are new, others provide new explanations for known phenomena.

- (1a) The number of firms is *negatively* related to *benefiting* possibilities within firms.
- (1b) The number of firms is *positively* related to the ratio of general *learning* possibilities over firm-specific learning possibilities.
- (1c) The number of firms is *negatively* related to the speed of *technological progress* in a given technological trajectory (first mover advantage).
- (1d) The number of firms is a *positive concave* function of the *number of workers* in the industry.
- (2a) Entry and exit of firms are *positively* correlated in time, after the early evolution of the industry when net entry is still large.
- (2b) Entry and exit rates in industries (as a percentage of the number of workers) in which there is little or no technological progress will *continue to be positive* as the selection process continues.
- (3) The *industry structure* of a more mature industry will be characterized by a coexistence of a handful of *large generalists*, some *small generalists* and quite a few *specialists*, the number of these dependent upon the number of productive talent vector elements.
- (4a) *Mobility* of workers between firms is *negatively* related to learning possibilities within firms (social reproduction).
- (4b) *Spin-offs* of the most talented workers in organizations are *positively* related to learning possibilities, while it does not matter for the less talented workers.
- (5) The *retirement* of an employer is likely to lead to spin-offs or even being taken over in case the learning possibilities were *low*, leaving the most talented employee clearly less productive than the former employer.
- (6a) Individuals with relatively *low talents* (in either dimension) are more likely to join *large corporations* than small firms.
- (6b) The average productivity (and talents) of employees of *small generalists* are higher than that of *small specialists*.
- (7a) Relatively *specialized* organizations will have more difficulty attracting more specialist than less specialist individuals.
- (7b) The employers with the *smallest productivity* (overall talents) that will still be able to run a firm will be *very specialized*.
- (8) *Large corporations* have more readily replacement of dysfunctional leaders than small companies (a larger pool of talents), and therefore have *higher survival chances* also in periods of technological turmoil.
- (9a) *Small* firms will attract individuals with a lower variety in the amount of talents but higher average level of productivity. The average productivity of the workers is high mainly because of the talents of the employees themselves.
- (9b) *Large* firms will attract individuals with a similar proportion of talents but in a wide amount of variety in the size thereof. The average productivity of the workers is high mainly because of the strong talents of the employer and the employees benefiting or learning from that.
- (10) In *migration of employees* between organizations, the less talented and less similar individuals will represent a higher proportion than the more talented and similar individuals.

- (11a) Mergers, acquisitions, take-overs and spin-offs are ways to restore the optimal equilibrium industry structure. *Take-overs* of complete companies are likely to be followed by *spin-offs* of parts of the firms that have been taken over.
- (11b) Spin-offs of firms share many characteristics of the original company and more so if the employees that spin-off have been in the company for a long time due to learning.
- (12) *Entry* of new firms and the *flexibility* of organizations are interrelated. In case employers are relatively slow in retiring when needed due to e.g. technological changes, there is ample opportunity for new firms to emerge.

Our model has a wide set of testable implications but also a list of limitations. First, we use very simple benefiting and learning rules (fixed parameters) being equal across firms and employees. It is obvious that some employees learn faster than other and that some organizations provide more efficient ‘learning architectures’ than others. Organizations may differ in the extent to which they foster or hinder the utilization and increase of talents. Second, the model has talents to be the sole determinant of productivity, thereby neglecting any role of capital investments. This limits the applicability of the model to professional service industries in which productivity is directly linked to individual capabilities. Third, we assume that there is absence of any entry and exit barriers save from being a worker in the industry (i.e. having talents higher than the cut-off point). These and other limitations are obvious candidates for further research.

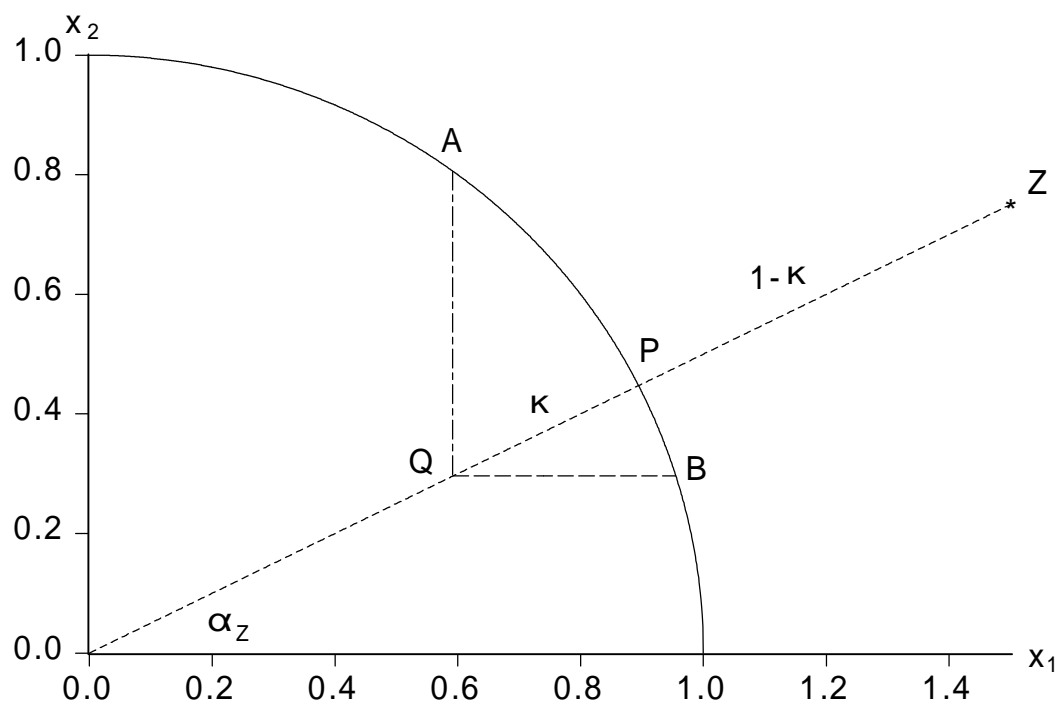
References

- Aron, D.J. (1988), Ability, moral hazard, firm size, and diversification, *RAND Journal of Economics* 19, 72-87.
- Baumol, W.J. (1990), Entrepreneurship: Productive, unproductive and destructive, *Journal of Political Economy* 98, 893-921.
- Bonaccorsi, A. and P. Giuri (2000), When shakeout doesn’t occur – The evolution of the turboprop engine industry, *Research Policy* 29, 847-870.
- Calvo, G.A. and S. Wellisz (1980), Technology, entrepreneurs, and firm size, *Quarterly Journal of Economics* 95, 663-677.
- Camacho, A. (1991), Adaption costs, coordination costs and optimal firm size, *Journal of Economic Behavior and Organization* 15, 137-149.
- Carree, M.A. and A.R. Thurik (2000), The life cycle of the U.S. tire industry, *Southern Economic Journal* 67, 254-278.
- Carroll, J.B. (1993), *Human Cognitive Abilities*, Cambridge: Cambridge University Press.
- Carroll, G., H. Haveman and A. Swaminathan (1992), Careers in organizations: an ecological perspective, in: D.L. Featherman, L.M. Lerner and M. Perlmutter (eds.), *Life-Span Development and Behavior*, 11: 112-144, Hillsdale, NJ: Lawrence Erlbaum.

- Carroll, G. and M. Hannan (2000), *The Demography of Corporations and Industries*, Princeton University Press.
- Dunne, T., M.J. Roberts and L. Samuelson (1988), Patterns of Firm Entry and Exit in U.S. Manufacturing Industries, *RAND Journal of Economics* 19, 495-515.
- Gardner, H. (1983), *Frames of Mind: The Theory of Multiple Intelligences*, New York: Basic Books.
- Garicano, L. (2000), Hierarchies and the organization of knowledge in production, *Journal of Political Economy* 108, 874-904.
- Gifford, S. (1993), Heterogeneous ability, career choice and firm size, *Small Business Economics* 5, 249-259.
- Gran, S. (1992), *A Course in Ocean Engineering*, Developments in Marine Technology, Vol. 8, Elsevier Science Publishers, Amsterdam. Internet edition: <http://www.dnv.com/ocean/>, alternatively: <http://www.math.uio.no/~ocean/course.htm>
- Hambrick, D., M. Geletkanycz and J. Frederickson (1993), Top management commitment to the status-quo: some tests of its determinants, *Strategic Management Journal* 14, 401-418.
- Haveman, H. (1995), The demographic metabolism of organizations: industry dynamics, turnover and tenure distributions, *Administrative Science Quarterly* 40, 586-618.
- Haveman, H. and L. Cohen (1994), The ecological dynamics of careers: the impact of organizational founding, dissolution and merger on job mobility, *American Journal of Sociology* 100, 104-152.
- Horn, J.L. (1985), Remodeling old models of intelligence, in B.B. Wolman (ed.), *Handbook of Intelligence. Theories, Measurements, and Applications*, New York: Wiley.
- Horn, J.L. and J. Noll (1994), A system for understanding cognitive capabilities: A theory and the evidence on which it is based, in D.K. Detterman (ed.), *Current Topics in Human Intelligence*, Vol.4: Theories of Intelligence, Norwood: Ablex.
- Johnson, N.L., S. Kotz and N. Balakrishnan (?), *Continuous Univariate Distributions*, Volume 1, Second Edition, John Wiley & Sons (New York), p.456-461.
- Jovanovic, B. (1994), Firm formation with heterogeneous management and labor skills, *Small Business Economics* 6, 185-191.
- Jovanovic, B. and G.M. MacDonald (1994), The life cycle of a competitive industry, *Journal of Political Economy* 102, 24-52.
- Kihlstrom, R.E. and J.J. Laffont (1979), A general equilibrium entrepreneurial theory of firm formation based on risk aversion, *Journal of Political Economy* 87, 719-748.
- Klepper, S. (1996), Entry, exit, growth, and innovation over the product life cycle, *American Economic Review* 86, 562-583.
- Klepper, S. (1997), Industry life cycles, *Industrial and Corporate Change* 6, 145-181.
- Klepper, S. and E. Graddy (1990), The evolution of new industries and the determinants of market structure, *RAND Journal of Economics* 21, 27-44.
- Klepper, S. and K.L. Simons (2000a), The making of an oligopoly: firm survival and technological change in the evolution of the U.S. tire industry, *Journal of Political Economy* 108, 728-760.
- Klepper, S. and K.L. Simons (2000b), Dominance by birthright: Entry of prior radio producers and competitive ramifications in the U.S. television receiver industry, *Strategic Management Journal* 21, 997-1016.
- Kremer, M. (1993), The O-ring theory of economic development, *Quarterly Journal of Economics* 108.
- Lawrence, B.S. (1997), Opening the black box of organizational demography, *Organization Science* 8, 1-22.
- Lucas, R.E. (1978), On the size distribution of business firms, *BELL Journal of Economics* 9, 508-523.

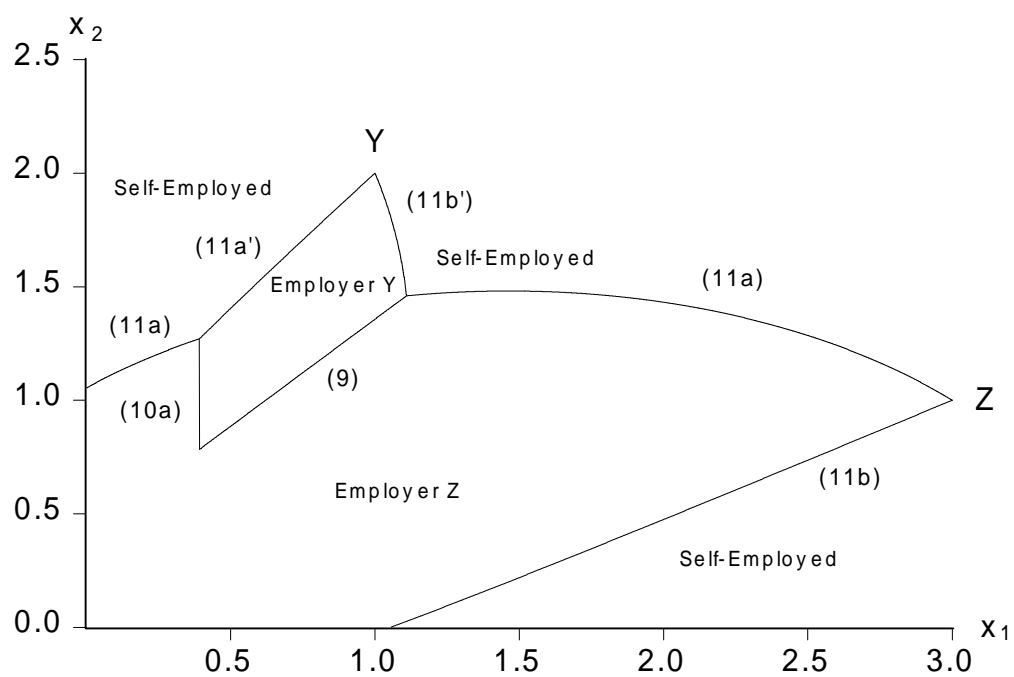
- Murphy, K.M., A. Shleifer and R.W. Vishny (1991), The allocation of talent: implications for growth, *Quarterly Journal of Economics* 106, 503-530.
- Pfeffer, J. (1983), Organizational demography, *Research in Organizational Behavior* 5, 299-357.
- Pfeffer, J. and G. Salancik (1978), *The External Control of Organizations: A Resource Dependence Perspective*, New York: Harper and Row.
- Piore, M.J. and C.F. Sabel (1984), *The Second Industrial Divide: Possibilities for Prosperity*, Basic Books (New York).
- Rosen, S. (1982), Authority, control, and the distribution of earnings, *BELL Journal of Economics* 13, 311-323.
- Shaw, M.E. (1976), *Group Dynamics: The Psychology of Small Group Behavior*, McGraw-Hill (New York).
- Smith, K.G., K.A. Smith, J.D. Olian, H.P. Sims jr, D.P. O'Bannon and J.A. Scully (1994), Top management team demography and process: The role of social integration and communication, *Administrative Science Quarterly* 39, 412-438.
- Sorensen, J. (2000), The ecology of organizational demography: managerial tenure distributions and organizational competition, *Industrial and Corporate Change* 8, 713-744.
- Sternberg, R.J. (1988), *The Triarchic Mind: A New Theory of Human Intelligence*, New York: Viking.
- Swaminathan, A. (1995), The proliferation of specialist organizations in the American wine industry, 1941-1990, *Administrative Science Quarterly* 40, 653-680.
- Szymanski, D.M., L.C. Troy and S.G. Bharadwaj (1995), Order of entry and business performance: An empirical synthesis and reexamination, *Journal of Marketing* 59, 17-33.
- Thurstone, L.L. (1938), *Primary Mental Abilities*, Chicago: Chicago University Press.
- Wagner, W.G., J. Pfeffer and C.A. O'Reilly III (1984), Organizational demography and turnover in top management groups, *Administrative Science Quarterly* 29, 74-92.
- White, H. (1970), *Chains of Opportunity: System Models of Mobility in Organizations*, Cambridge, MA: Harvard University Press.
- Witteloostuijn, A. van (1998), Bridging behavioral and economic theories of decline: organizational inertia, strategic competition and chronic failure, *Management Science* 44, 501-520.

Figure 1: Choosing to become (self-)employed



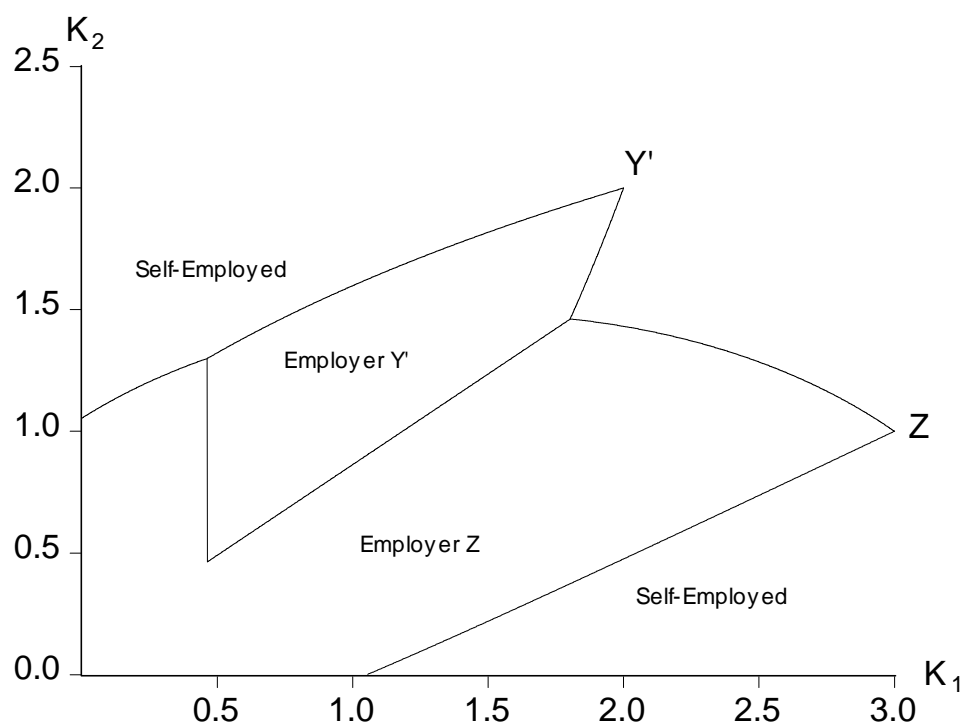
Note: Individuals A and B with talents on the unit circle are indifferent between becoming self-employed and joining firm Z. Individuals on the unit circle in between A and B will join firm Z.

Figure 2: Choosing between employers I



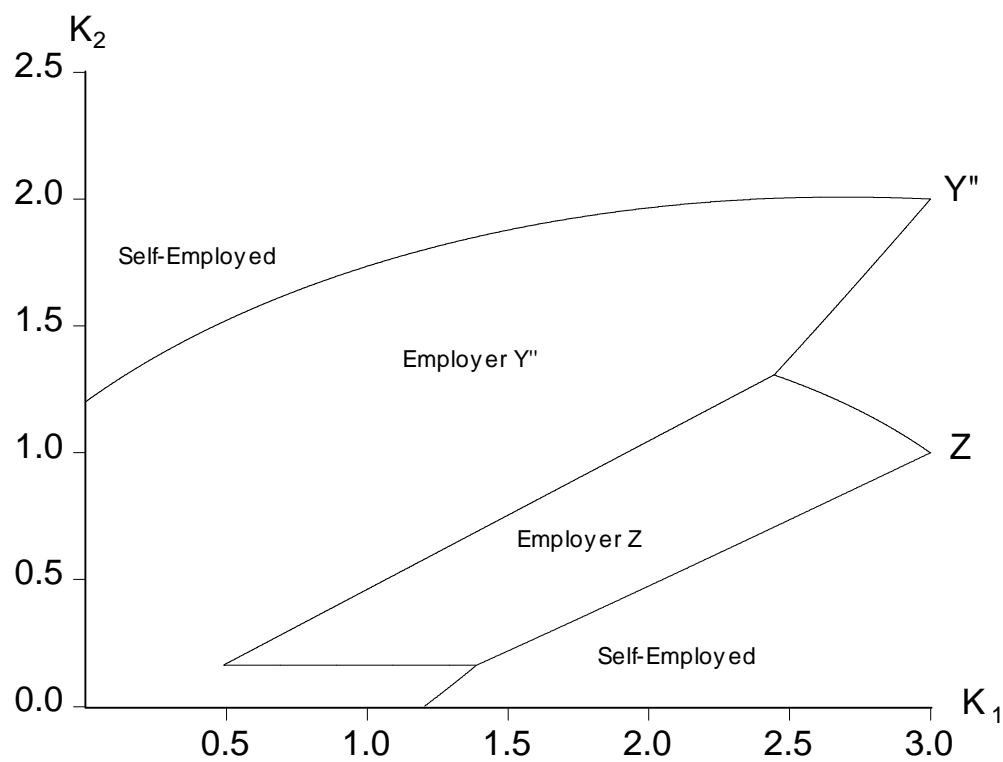
Note: Firm Y is located at (1,2) and firm Z is located at (3,1). Kappa is one third. The value of x_1 below which all individuals prefer Z above Y is 0.392.

Figure 3: Choosing between employers II

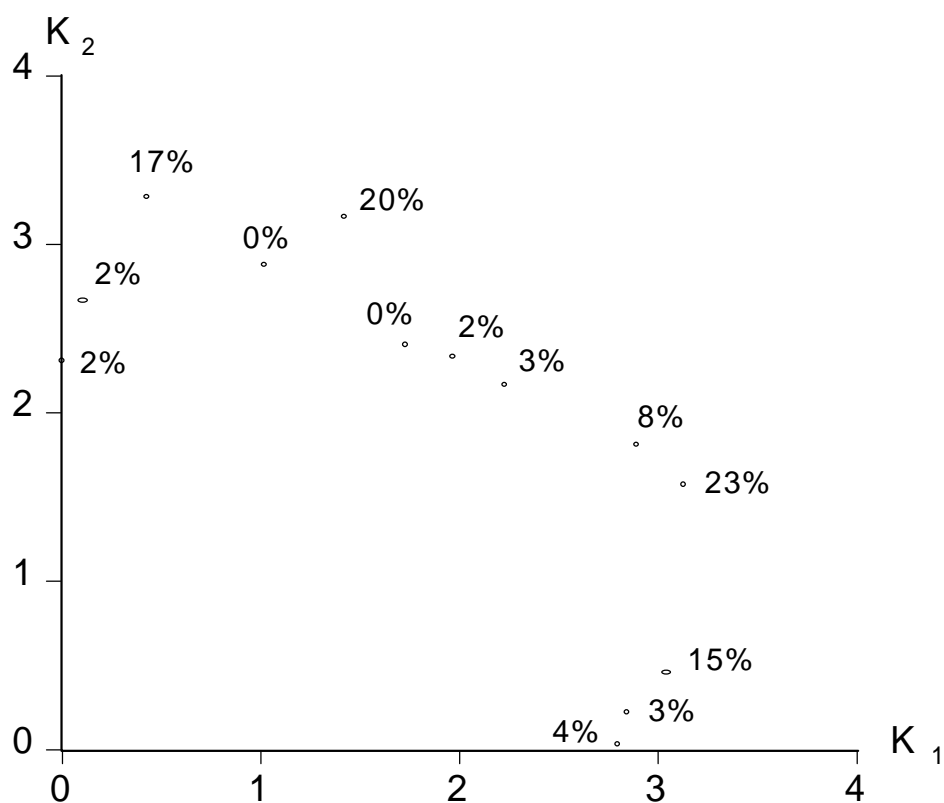


Note: Firm Y' is located at (2,2) and firm Z is located at (3,1). Kappa is one third.

Figure 4: Choosing between employers III

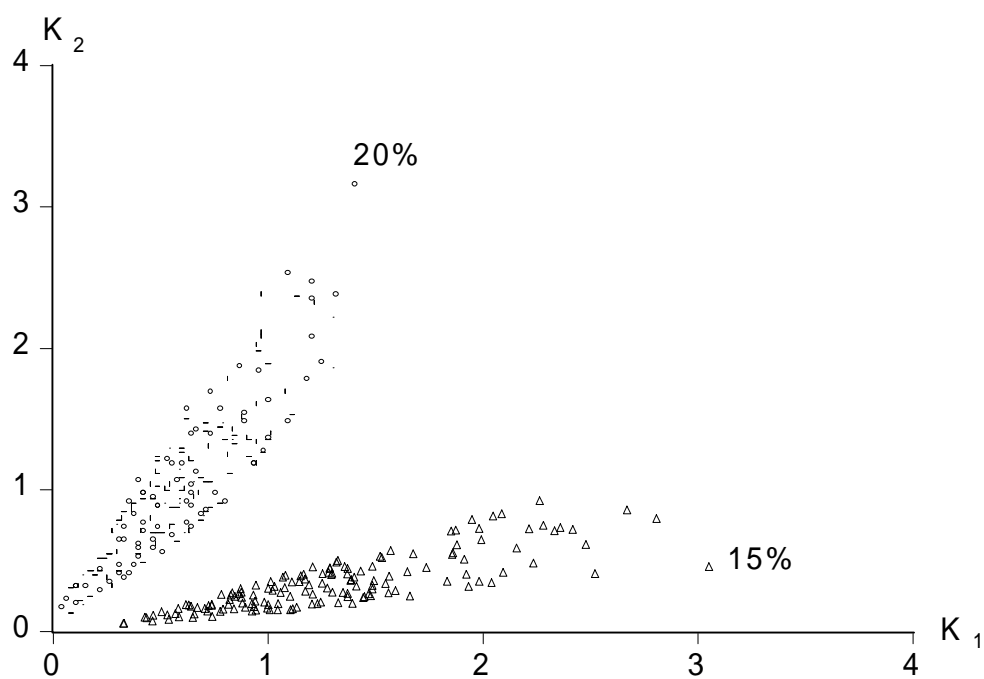


Note: Firm Y'' is located at (3,2) and firm Z is located at (3,1). Kappa is one third.

Figure 5: Thirteen firms in a sample of $N=1000$ 

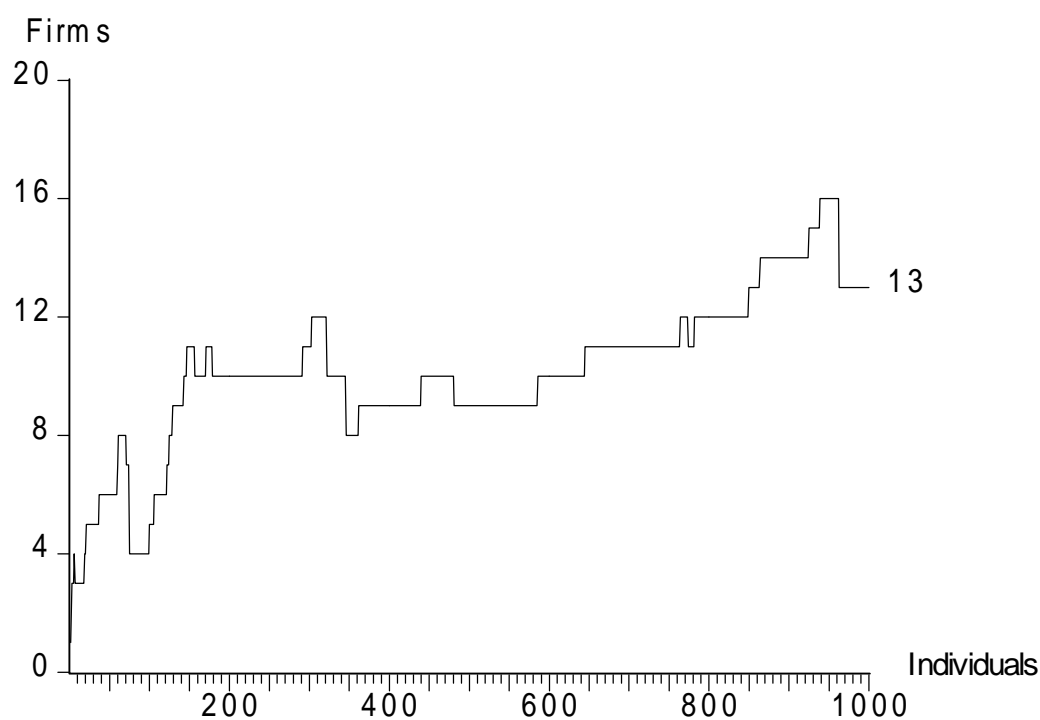
Note: This simulation of 1,000 individuals returns an equilibrium configuration of thirteen firms of which the talent vectors are small circles and the market shares in terms of employees are given next to them. Kappa is one third.

Figure 6: The employees of two large firms



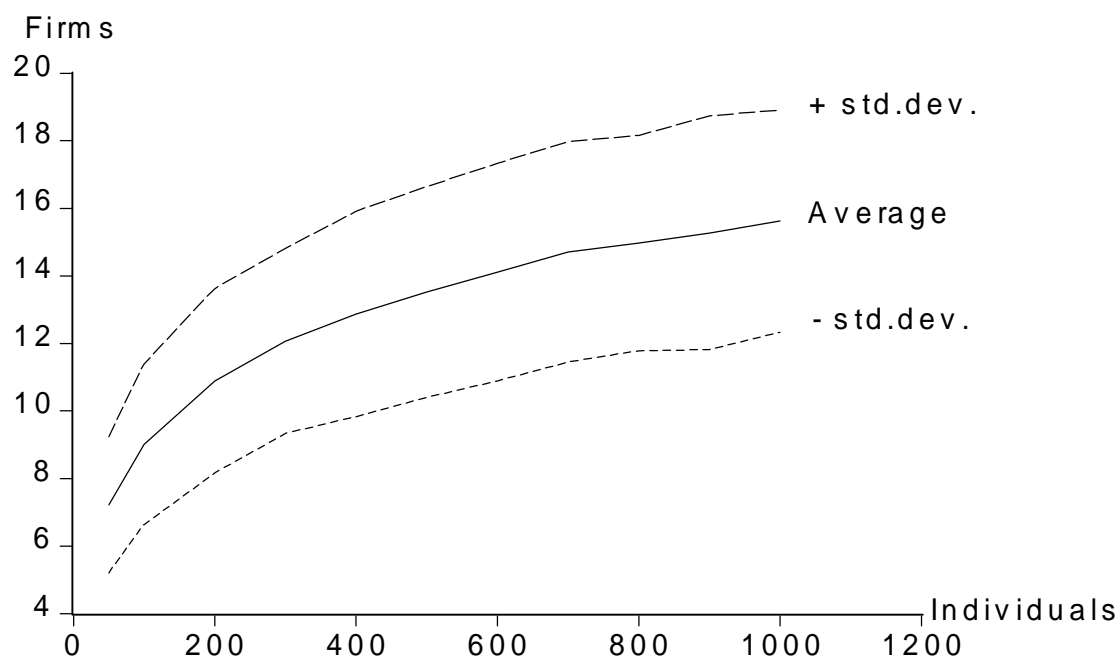
Note: see Figure 5. For two firms the talent vectors of the employees are presented.

Figure 7: A simulation of number of firms



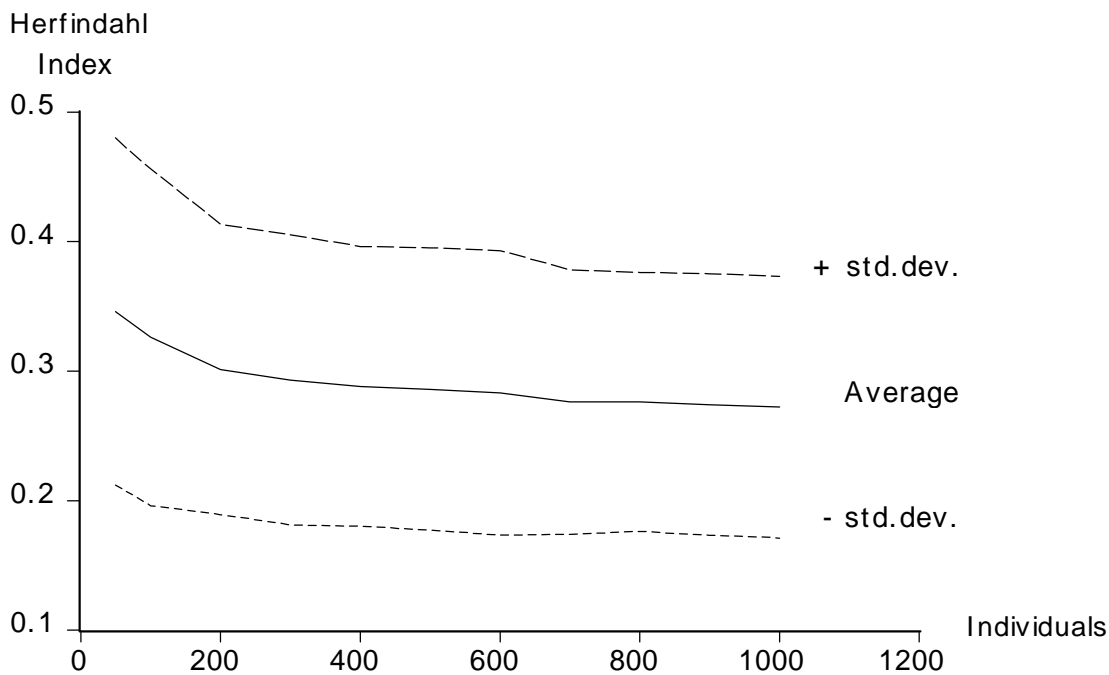
Note: See Figures 5 and 6. The equilibrium number of firms with an increasing numbers of individuals is given. When there are 1,000 individuals present the equilibrium number is thirteen.

Figure 8: Average number of firms in simulations



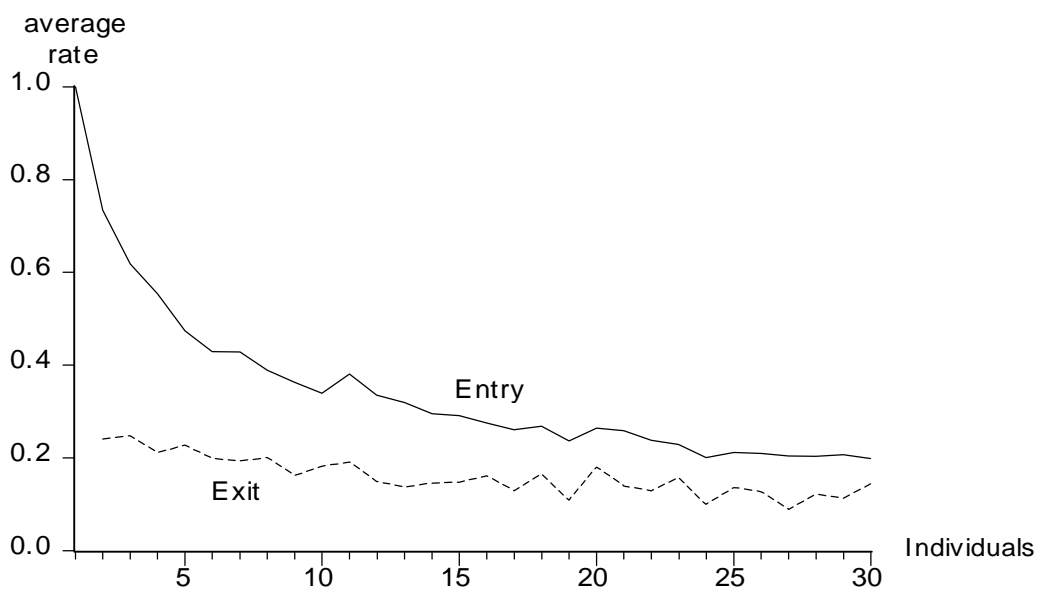
Note: After 1,000 simulations of 50, 100, 200, ..., 1,000 individuals, the average and standard deviation of the equilibrium numbers are computed. The figure presents the average and one standard deviation above and below. Kappa is one third.

Figure 9: Average Herfindahl Index in simulations



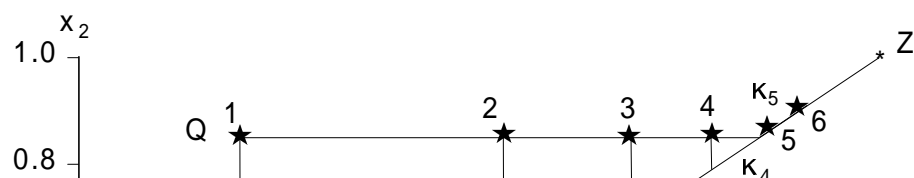
Note: After 1,000 simulations of 50, 100, 200, ..., 1,000 individuals, the average and standard deviation of the equilibrium Herfindahl Index are computed. The figure presents the average and one standard deviation above and below. Kappa is one third.

Figure 10: Average entry and exit rates for the first 30 individuals



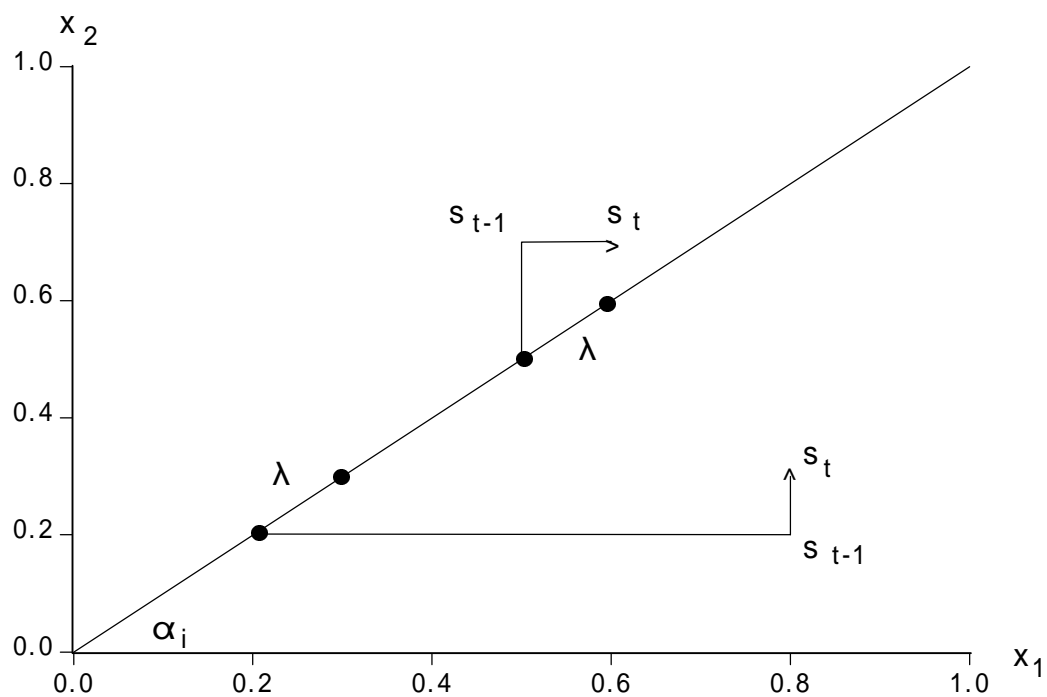
Note: After 1,000 simulations of the first 30 individuals the average rates of entry and exit per new worker were computed. The straight line corresponds to the average rate of entry, viz. the rate at which new workers in the industry start up new firms. The broken line corresponds to the average rate of exit, viz. the average number of employers that become employee. Kappa is one third.

Figure 11: Learning and converging, first variant



Note: Six periods of the development of knowledge (learning) for an employee is presented, given that the employer does not extend his or her knowledge. Kappa is one third.

Figure 12: Learning by doing, second variant



Note: For two individuals s the shift in the talent vector from period $t-1$ to period t is presented. The learning parameter is equal to lambda.

Appendix: Approximating the equilibrium number of firms

In this appendix we derive an approximation for the equilibrium number of firms as a function of the benefiting parameter κ and the number of individuals N . The approximation is derived by considering the l_1 -firm to be on the horizontal axis ($\alpha_1 = 0$), the l_2 -firm to be on the vertical axis ($\alpha_2 = \pi/2$), the l_3 -firm to have the angle at which an individual with talent length \sqrt{s} (hence, productivity when self-employed of s) is indifferent between being employed by the l_1 -firm and being self-employed, etc. That is, the firms with odd numbers when ordered in talent vector length cover the small angles, while the firms with even numbers cover the large angles. The approximation procedure is to add firms till the complete spectrum is covered. For above and below the line individuals we have:

$$\alpha_s = \arccos\left(x_{1i} \frac{1/l_i - \kappa/\sqrt{s}}{1-\kappa}\right) \quad i = \text{even} \quad \wedge \quad \alpha_s = \arcsin\left(x_{2i} \frac{1/l_i - \kappa/\sqrt{s}}{1-\kappa}\right) \quad i = \text{odd}$$

The critical corners for the odd and even are then:

$$\alpha_{2p+1} = \arccos\left(\prod_{i=1}^p \frac{1 - \kappa l_{2i-1} / \sqrt{s}}{1 - \kappa}\right) \quad i = \text{odd} \quad \wedge \quad \alpha_{2p+2} = \arcsin\left(\prod_{i=1}^p \frac{1 - \kappa l_{2i} / \sqrt{s}}{1 - \kappa}\right) \quad i = \text{even}$$

The question is at which p the corners will meet. It should be when the term between brackets is about $1/\sqrt{2}$. We search for the value of p so that $\alpha_{2p+1} > \alpha_{2p+2}$. The estimated number of firms is then $2p$. It can be derived that the value of p is that for which for the first time:

$$\sum_{i=1}^p \ln \left(\frac{1 - \kappa \sqrt{\frac{\ln(N) - \ln(2i-1)}{\ln(N) - \ln(2p-0.5)}}}{1 - \kappa} \right) < -0.5 \ln(2)$$

A simple computer program can be used to determine p . The results of one thousand iterations of equilibrium with different N and κ are presented in Table A1. The approximation appears

to work satisfactorily. There is a some overestimation for low values of κ (probably talented individuals in the middle in the simulation).

Table A1: The approximation of the equilibrium number of firms

κ	N	APPROX	NOF			min	max	HI		APROD	
			mean	medi	std			mean	std	mean	std
0.1	100	24	18.6	18	3.1	9	31	0.126	0.035	2.20	0.22
0.2	100	14	12.7	13	2.7	5	23	0.202	0.066	2.52	0.27
0.3	100	10	9.8	10	2.4	3	18	0.291	0.111	2.99	0.38
0.4	100	8	7.7	8	2.2	2	15	0.395	0.156	3.56	0.51
0.5	100	6	5.9	6	2.0	1	12	0.517	0.199	4.26	0.73
0.1	500	38	30.7	31	4.3	19	44	0.103	0.026	2.33	0.11
0.2	500	22	19.5	19	3.6	9	32	0.177	0.055	2.82	0.16
0.3	500	14	14.7	15	3.0	7	25	0.257	0.095	3.49	0.27
0.4	500	10	11.7	12	2.9	4	22	0.348	0.136	4.29	0.42
0.5	500	8	9.1	9	2.6	2	19	0.463	0.191	5.30	0.65
0.1	2000	52	42.5	42	5.4	26	61	0.094	0.024	2.44	0.06
0.2	2000	28	25.9	26	4.4	15	46	0.165	0.049	3.09	0.12
0.3	2000	18	19.4	19	3.6	8	31	0.231	0.080	3.91	0.22
0.4	2000	14	15.0	15	3.4	5	27	0.325	0.123	4.95	0.38
0.5	2000	10	11.8	12	3.1	2	22	0.440	0.175	6.18	0.58

Note: 'medi' stands for median and 'std' stands for standard deviation. The column with APPROX gives the approximate number of firms.