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TRANSITIONAL DYNAMICS OF ENTRY IN PRESENCE OF  
TECHNOLOGICAL KNOWLEDGE CHANGE

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**Abstract**

Industrial structures constantly change over time. The paper presents a model of industrial evolution in a dynamic setting. The model explicitly recognises that technologies continuously change their nature and, accordingly, the technological knowledge associated with them evolves. The aim of the paper is to explore, in an equilibrium model, the transitional dynamics of entry when the scientific and technological knowledge underlying the dominant technology of the industry evolves. We believe that the process of technological knowledge accumulation contributes in shaping the evolution and dynamics of market structures. Recognising, then, that the technological knowledge is subject to continuous changes, we explore the consequences of such shocks on the entry process of new firms and on the evolution of the existing stock of knowledge, distinguishing between the effects of competence-broadening and competencenarrowing shocks. We then propose an explanation of the industry life cycle evolution in terms of succession of changes affecting the existing technological knowledge.

We argue that the reasons why some industries do not follow the industry life cycle path might be explained by our model. In absence of an exogenous shock that leads to a competence-narrowing technological change, the evolutionary path of the industrial structure is not likely to follow the industry life cycle pattern. We also study the effects of some government policies aimed at improving the degree of competition in the market as well as the level of technological knowledge in the industry. Finally, we explore the transitional dynamics of the model in the presence of agglomeration effects in knowledge development across firms.

**Keywords:** Industrial Dynamics – Technological Change – Knowledge – Dynamic Systems

**JEL classification:** L10, C61, O3

## **1. Introduction**

Industries are in a constant state of flux over time. One of the main factors that contributes to this industrial turbulence is found in the process of entry of new firms into the markets. Then, when examining industries' evolution, we also need to explicitly recognise that technologies continuously change their nature and, accordingly, the scientific and technological knowledge associated with them evolves.

Numerous studies have focused their attention on the role of technological change in shaping the evolution of industrial sectors. In particular, the relationship between technological evolution and the process of entry of new firms has been analysed. The aim of our paper is to explore, in an equilibrium model, the transitional dynamics of entry when the scientific and technological knowledge underlying the dominant technology of the industry evolves. We believe that the process of technological knowledge accumulation contributes in shaping the evolution and dynamics of market structures.

We study a model in which the technological knowledge evolves over time, according to an autonomous process of growth and to the contribution of new entering firms into the market. The entry phenomenon is seen as a competence-driven occurrence: in order to find entry profitable, new firms need to embody the existing required stock of knowledge, as well as new knowledge and expertise. We analyse a non-linear differential model in which we focus the attention on the transitional dynamics towards the steady state. Recognising, then, that the technological knowledge is subject to continuous changes, we explore the consequences of such shocks on the entry process of new firms and on the evolution of the existing stock of knowledge.

We also study the effects of some government policies aimed at improving the degree of competition in the market as well as the level of technological knowledge in the industry. Finally, we explore the transitional dynamics of the model in the presence of agglomeration effects in knowledge development across firms.

## **2. Does Technological Change Affect the Pattern of Entry?**

Since the last century, the significance of technological innovations and their discontinuous character in determining the evolutionary pattern of industries has been clearly recognized. In "The Theory of Economic Development" (1934), Joseph Schumpeter was one of the first to stress the importance of the study of the dynamic aspects of industries' evolution. The Austrian economist extensively analysed the process of economic change. While recognising the existence of periods in which incremental technological improvements and refinements characterise market evolution, he mainly focused his attention on the role played by technological discontinuities and entrant firms in shaping the evolutionary dynamics of industries.

According to the Schumpeterian view, as expressed in “The Theory of Economic Development”, it is possible to understand the economic process of industrial change as a continuum of periods of dominance of a certain technology and its related incumbent firms, followed by a new technological breakthrough and the entry of new firms displacing the old ones. The main determinant of the process of entry has to be found in the opportunities opened up by technological innovations and by the innovative capabilities of new entrants, that are able to bring “something new” into the market. In this view, firms that enter the market bring some new knowledge and capabilities: the process of entry is competence-driven.

A more particular distinction that has been widely used in the economic literature uses the terms “incremental innovation” and “radical innovation”. In what follows, we consider the first to be an incremental refinement of the existing technology, while we consider a “radical innovation” to be a revolutionary breakthrough, or better, a technological discontinuity, caused by the introduction to an industry, usually by new firms, of a technology of a superior order to the old one. Radical innovations represent, then, the turning point of industries’ evolution: their effects are widely different from those of incremental innovations, unsettling the market structure and the relative strength of each firm in the industry. With regard to market structures, it is possible to recognize that often the evolutionary pattern of industries is characterized by cycles of incremental technical change punctuated by technological breakthroughs that open up a new era. Some authors (Windrum, 1999) name this situation “technological succession”. These discontinuities appear at any point in time, at irregular intervals, in the economic cycle. Usually these periods of technological revolution are associated with the entry of new firms to the industry, bringing into the market the new expertise, skills, routines, competencies and heuristic problem-solving procedures that are embodied into the new technology. According to this approach, it is with the emergence of a technological breakthrough that the process of entry of new firms has the opportunity to diffuse and prosper. However, it is important to stress that not all types of technological discontinuity are accompanied by the emergence and dominance of new firms. With regard to this, Tushman and Anderson (1986) identified the difference between “competence-enhancing” and “competence-destroying” technological change.

### ***2.1 Competence-Enhancing and Competence-Destroying Technological Change***

Distinguishing between the concepts of competence enhancement and competence destruction is of particular interest for the study of the evolution of market structures and of the emergence of new firms in the industry. Competence-destroying discontinuity is associated with the manifestation of new knowledge and competencies deriving from the utilisation of the new technology, which differ from the expertise previously developed by the incumbents, and from the know-how linked to existing products and production processes. In contrast, competence-

enhancing technological change further improves upon the existing technology, building on the experience, skills and competencies that have already been accumulated by the incumbent firms over time.

It is clear that, in the former case, the effect on the market structure is extremely striking and disruptive. New firms enter the market embodying the new knowledge and technological expertise. They are revealed to be more effective and faster in the introduction of the new technology, while the incumbent firms demonstrate a sort of lock-in into the old procedures and existing technology. The existing technology is then rendered obsolete by the entrance of the new firms. The result is the emergence and dominance of a new population of entrants, that displace the existing firms from the market. This does not occur in a situation of competence-enhancing technical change. In such a situation, incumbents can rely on their experience and accumulated knowledge base, which turn out to be important and necessary in the process of exploiting the new technology and competencies. Anderson and Tushman (1990) analyse the cases of different industries (cement, container glass, flat glass and mini-computer industries) in which competence-destroying and competence-enhancing technological discontinuities shaped the evolution of the market structure according to their theory.

Further to these concepts, according to Henderson and Clark (1990), and Henderson (1993), there are several cases in which the disruptive effects on firms' competencies are due to innovations and changes in the so-called "architecture" of the products. In these situations, what changes is not the technological base but the relationships and the hierarchy of the components and parts that make the product. These kinds of change can have a deep impact on the evolution of the market structure, as happened in the case of photolithographic machines. If the incumbent firms focus on the existing architectures, underrating the importance of the new ones, then it could happen that new firms come into the market and succeed in dominating it.

The argument of Henderson and Clark is closely linked to the informational and knowledge changes inside a firm. In fact, important concepts strictly related to the architecture of the product include the knowledge reflecting the problem-solving capability and the managerial procedures of the architecture itself. Examples include implicit and explicit informational channels inside the organization, procedural routines, and the communication practices among researchers and engineers. In cases of emergence of a new architecture, established firms tend to merely modify and not radically change their procedures, such that they end up in a situation of disadvantage compared to the newcomers that calibrate their structure on the new architecture.

### **3. The Model**

The relationship between entry and change in technological knowledge has been extensively examined in the economic literature both from an empirical and from a

theoretical point of view. As we have discussed above, technological change might have various and different effects on the type and extent of entry and on industries' evolution, depending on its nature and on the market characteristics.

Datta and Dixon (2002)<sup>1</sup> propose a model in which the flow of entry determines the cost of entry, which is linked to the net present value of incumbent firms through an arbitrage condition. The model explores the relationship between changes in technology parameters and the stock market valuation of firms and shows that there can be a non-monotone transition dynamic of share price even if the underlying technology is improving. Here, we also explore an equilibrium model for industry entry dynamics and technological change in which we define a cost of entry linked to an arbitrage condition, and we also examine the role of changes in technology. But we expressly depart from their model because we want to focus our analysis on the recognition that technology is strictly linked to knowledge, and on the role of technological change with regards to current competencies and knowledge, while explicitly considering that knowledge evolves. In particular, although in the spirit of the Datta and Dixon analysis, we set up the model differently, by structuring the evolution of what we define "technological knowledge", considering the role of innovative entrants and endogenising the cost of entry.

The aim of our model is to capture the relationship between entry and technological knowledge evolution, by analysing the role of innovative entrants and the consequent evolution of the industry. In our model, the entry phenomenon is typical of the process of change in the market in the sense that entry is strictly related to the advancement of the scientific-technological knowledge of the industry. In other words, we offer an analysis of industry turbulence as a phenomenon driven by the evolution of knowledge in a dynamic setting, and from this point of view, we clearly diverge from traditional models that depict entry as a phenomenon that finds its explanation purely by referring to the adjustment process of extra-profits towards the equilibrium. The mechanism at work in our setting is that knowledge evolution affects the process of entry, and correspondingly entry is seen as competence-driven occurrence: an arbitrage condition governs the dynamics of the model, but entry, in order to be profitable, need to be accompanied by a required level of technological knowledge and competencies.

The model is expressed in terms of structural dynamic equations. The variables under examination are the number of firms and the level of technological knowledge. We study these variables in aggregate terms. In particular we assume that there is a population of firms in an industry, whose number we denote as  $n$ , characterised by a given type of technology, that is linked to a given type of technological knowledge, called  $S$ . The basic model shows the transitional dynamics of  $n$  towards its steady-state level when the technological knowledge evolves over time. We, then, show the consequences of a change in the technological knowledge on the dynamic path of  $n$ .

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<sup>1</sup> Datta B., Dixon H. (2002), "Technological Change, Entry and Stock Market Dynamics: an Analysis of Transition in a Monoplistic Economy", *American Economic Review, Papers and Proceedings*, 92, 231-235

Recognising that technologies follow an evolutionary path or trajectory during which developments, improvements and modifications take place, allows us to shed some light also on the structural evolution of the industry.

It is clear, then, that the model is not established on micro-economic foundations, and we leave this further development of the model for future research.

The basic assumption about firms and technology requires that each firm of the population embodies the given type of what we define “technological knowledge” or “technological competencies”,  $S$ , that is, a specific knowledge that is linked to a particular technology. More precisely, we include in this term, “knowledge”, know-how and expertise that are technology-specific that ultimately lead to industrial applications like products and technological artifacts. A change in the basic characteristics of the existing technology, then, leads to changes in the corresponding type of knowledge and ultimately in the active population of firms embodying this existing knowledge. We specifically analyse this process in the transitional dynamics of the model.

Our model aims at explaining the process of entry in the presence of technological changes in a cumulative industry, in the sense that the technological knowledge constantly accumulates over time: when new firms enter into the industry, they bring new technological knowledge that adds to the existing stock. In the immediately following period, then, the aggregate knowledge in the industry increases as a consequence of the process of entry. It follows that new entrants embody more and different technological expertise that let them produce a different variant of the product and enter the market. This is possible only if the entrant firms firstly acquire a share of the existing stock of knowledge of the industry (in the model we indicate this share with  $\gamma$ ). This situation lasts for one period. After the period of entry, the new technological knowledge becomes publicly available.

The model does not explicitly focus on the explanation of the process of exit: we do not formalise a selection process. The decrease in the number of firms needs to be interpreted as the adjustment of the number of firms towards the sustainable level in the industry.

### ***3.1. Set-up of the Model***

#### *3.1.1. The Cost of Entry*

The model assumes that the cost of entry at time  $t$  is proportional to the existing stock of knowledge in the industry. The explanation of this assumption is that firms need to endow themselves with a minimum level of the existing knowledge in order to be effective and competitive in the market. Consequently, in accordance with the evolution of  $S$ , the cost of entry varies in time: it is endogenously determined and it evolves with the key variables in the model. Formally, if we indicate with  $C$  the cost of entry and with  $S$  the level of the existing aggregate knowledge at time  $t$  in the industry, we have:

$$(1) \quad C = \gamma S$$

where  $\gamma$  is the share of the existing aggregate knowledge  $S$  that firms have to acquire in order to be able to enter the market and have the necessary competencies to effectively produce. Note that  $\gamma < 1$ , such that  $(1-\gamma)$  represent a type of positive externalities extending from incumbent firms towards newcomers, in the sense that part of the aggregate knowledge does not need to be acquired in order to enter. The intuition behind this might be explained as a second-mover advantage of the newcomers that are able to discern, and consequently cut off, that part of aggregate old knowledge that is superfluous for being competitive in the industry. Alternatively, we might interpret  $(1-\gamma)$  as the part of old knowledge that new firms do not need because they embody and bring into the industry new competencies as replacements.

### 3.1.2. Arbitrage Condition

The opportunity cost of funds is given by the fixed return  $r$  of government bonds. The following arbitrage condition that determines the relationship between the return on investing a dollar in setting up a new firm and  $r$  must hold:

$$(2) \quad \frac{\Pi}{C} + \frac{\dot{C}}{C} = r$$

The left-hand term represents the number of firms per dollar times the firm's profits plus the change in the cost of entry, while the right-hand side reflects the returns on bonds. The change in the cost of entry  $\dot{C}/C$  denotes when entry is becoming cheaper (if it is  $< 0$ ), dissuading immediate entry, or when it is becoming more expensive (if it is  $> 0$ ), encouraging entry at time  $t$ .

### 3.1.3. Evolution of the Technological Knowledge

Acknowledging that technologies and related knowledge evolve over time, we structure the evolution of the industry's level of the "technological knowledge"  $S$  following the widely accepted argument that technological advance is a function of R&D, and that there exist diminishing returns to R&D. Quoting Nelson *et al.* (1997), "*An industry might initially have rich technological opportunities... Nonetheless, it seems inevitable that sooner or later R&D based on the initial stock alone would experience diminishing returns. In the absence of processes that renew technological opportunities, the opportunities would become mined out...*". Abstracting from the specification of any direct link between R&D and the technological advance, we specify, a decreasing return functional form for describing the evolution of  $S$ . Moreover, we focus also on the role that new firms have in determining the development and growth of the general level of competencies in the

industry, contributing in this way to the revitalisation and evolution of the competencies. *“The history of technical advance in these fields (pharmaceuticals and computers) reveals a periodic rejuvenation from outside the industry of new scientific understanding, materials, components... Industries where R&D intensity and technological opportunity remain high must be receiving a high rate of flow of new technological opportunities to make up for those that are being mined out”* (Nelson *et al.*, 1997).

One basic assumption of the model is that  $S$  is technology-specific. We are specifying here, first of all, the role of new entrants in determining this process or opportunities renewal (function  $h$ ), and secondly the autonomous development of knowledge (representing the evolution of the opportunities, applications and expertise linked to the corresponding technology) characterised by a decreasing returns function  $f$ , with  $f' > 0$ ,  $f'' < 0$ , and by an obsolescence effect  $\delta$ :

$$(3) \quad \dot{S} = f(S) + h(\dot{n})$$

More precisely, we assume the functions above to take the following special forms<sup>2</sup>:

$$(4) \quad \dot{S} = g S^\theta - \delta S + \phi \dot{n}$$

where  $\theta < 1$ . In other words, we are saying that in the absence of entry ( $\dot{n} = 0$ ) the level of knowledge is assumed to grow at the rate  $\dot{S} = g S^\theta$ , minus the obsolescence effect given by  $\delta S$ , where  $\delta < 1$ . This process might be seen as representing the general societal advance in the technological expertise and applications (that we interpret as refinements in the existing know-how and advancements outside the firms, i.e. at universities, research centres, scientific parks...), and the corresponding process of obsolescence. On the other hand, in the presence of entry we have another positive contribution to the development of knowledge given by the newcomers. In this way, we aim at modelling new firms as depositories of new knowledge, and entry is well represented by a “competence-driven process”. In fact, new technology systems might originate in *“[...] small firms started up by entrepreneurs with advanced university training in specialized areas, such has as been seen in microelectronics and biotechnology, or revolutionary new ideas as in the case of Henry Ford”* (Freeman and Soete, 1997). It is important to note that it is not the number of firms (stock) that contributes to the advances in the stock of knowledge: the positive change in the number of firms present in the industry,  $\dot{n} > 0$ , is the cause<sup>3</sup>.

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<sup>2</sup> In Appendix 1 we propose a different functional form for  $f(S)$ .

<sup>3</sup> In the final section, we explore a version of the model in which the stock of firms also contributes to improvements in the stock of knowledge.

### 3.1.4. The Profit Function

Firms' profits are taken to be a function of both the level of knowledge  $S$  and the number of firms in the industry  $n$ :

$$(5) \quad \Pi = \Pi(S, n)$$

Having defined  $S$  as above, it is reasonable to specify profits as an increasing function in the level of knowledge, while the number of firms, raising competition, contributes negatively to the level of profits (Polo, 1993), such that:

$$\Pi_S > 0 \text{ and } \Pi_n < 0$$

The assumption that  $\Pi_S > 0$  implicitly defines a demand function that positively links the total quantity demanded in the industry to the quality improvements and new variants of the product (i.e. the demand elasticity with respect to quality and new variants is greater than one), that we assume to be positively related to the level of knowledge  $S$ . An alternative interpretation may be found in the fact that developments in the stock of knowledge lead to cost savings in the production function of the existing applications, and consequently to an increase in the aggregate level of profits in the industry.

In the further development of the model, to simplify the analysis we assume a linear functional form for  $\Pi$ . We also take into account the level of the tax rate,  $\tau$ , such that the profit function assumes the following specification:

$$(6) \quad \Pi = (1-\tau)(\alpha S - \beta n)$$

### 3.2. Derivation of the Model and Steady State Analysis

Putting together (1) and (2), we can rewrite the arbitrage condition as follows:

$$(7) \quad \frac{\Pi}{\gamma S} + \frac{\dot{S}}{S} = r$$

which gives:

$$(7\text{bis}) \quad \dot{S} = Sr - \frac{\Pi}{\gamma}$$

Equating (7bis) and (4) we obtain:

$$(8) \quad Sr - \frac{\Pi}{\gamma} = g S^\theta - \delta S + \phi \dot{n}$$

which gives:

$$(9) \quad \dot{n} = \frac{1}{\phi} \left[ Sr - \frac{\Pi}{\gamma} - g S^\theta + \delta S \right]$$

Combining the two equations (7bis) and (9), the following system of two first-order differential equations results:

$$(10) \quad \begin{cases} \dot{S} = Sr - \frac{\Pi}{\gamma} \\ \dot{n} = \frac{1}{\phi} \left[ Sr - \frac{\Pi}{\gamma} - g S^\theta + \delta S \right] \end{cases}$$

### 3.2.1. Steady State Analysis

Let us study the two structural equations of the model in the steady state. From (7bis), setting  $\dot{S} = 0$  we obtain:

$$S = \frac{\Pi}{r\gamma}$$

and substituting (6):

$$(11) \quad n = \frac{\alpha - \frac{r\gamma}{1-\tau}}{\beta} S$$

On the other hand, from (9), setting  $\dot{n} = 0$ , and substituting (6), we have:

$$\frac{1}{\phi} \left[ Sr - \frac{(1-\tau)(\alpha S - \beta n)}{\gamma} - g S^\theta + \delta S \right] = 0$$

which, solving for  $n$ , gives:

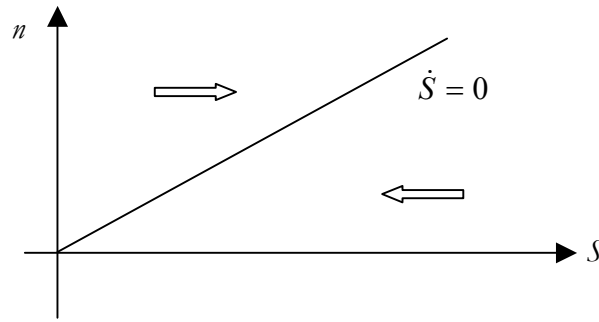
$$(12) \quad n = \frac{1}{(1-\tau)\beta} \left\{ \gamma g S^\theta - [r\gamma - (1-\tau)\alpha + \delta\gamma] S \right\}$$

In order to obtain a qualitative description of the properties of the solution of the system, we construct the phase diagram on the plane  $(S, n)$ . The loci of points on the plane  $(S, n)$  along which  $\dot{S} = 0$  and  $\dot{n} = 0$  are as follows:

- $\dot{S} = 0$

If:  $(1-\tau)\alpha > r\gamma$ , we obtain a positive sloped line (Figure 1), given by equation (11).

Figure 1: Loci of points on the plane  $(S, n)$  along which  $\dot{S} = 0$



- $\dot{n} = 0$

If:  $(1-\tau)\alpha < r\gamma + \delta\gamma$ , we obtain equation (9), which, after substituting (6) gives (12):

The first order condition gives:

$$\frac{\partial n}{\partial S} = \frac{1}{(1-\tau)\beta} \left\{ \gamma g \theta S^{\theta-1} - [r\gamma - (1-\tau)\alpha + \delta\gamma] \right\} = 0$$

which, solving for  $S$ , gives:

$$S = \left\{ \frac{\gamma g \theta}{\gamma r + \gamma \delta - (1-\tau)\alpha} \right\}^{\frac{1}{1-\theta}}$$

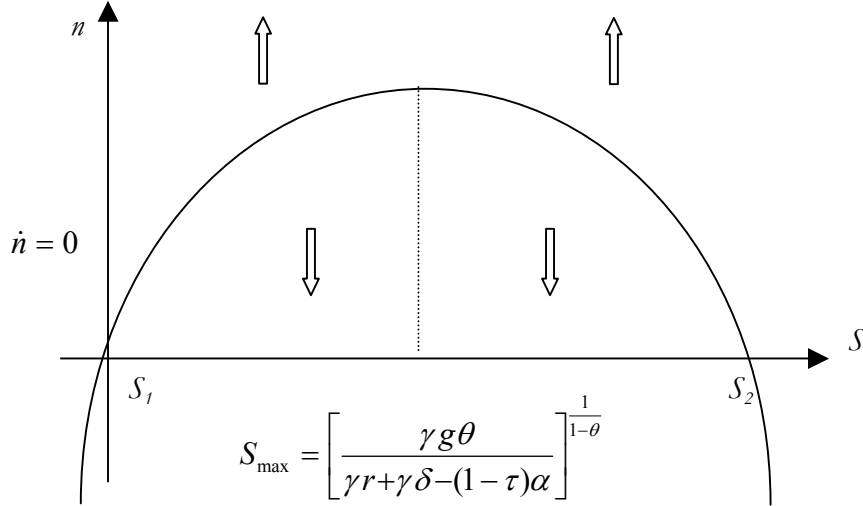
Since  $\theta < 1$ , when condition  $(1-\tau)\alpha < r\gamma + \delta\gamma$  holds, the second derivative:

$$\frac{\partial^2 n}{\partial S^2} = \frac{1}{(1-\tau)\beta} \theta(\theta-1)\gamma g S^{\theta-2} < 0$$

is clearly negative, denoting a maximum.

We can represent equation (12), i.e.  $\dot{n} = 0$ , in Figure 2.

Figure 2: Loci of points on the plane  $(S, n)$  along which  $\dot{n} = 0$



where the intersections with the horizontal axis are given by:

$$S_1 = 0 \text{ and } S_2 = \left[ \frac{\gamma g}{\gamma r + \gamma \delta - (1 - \tau) \alpha} \right]^{\frac{1}{1-\theta}}$$

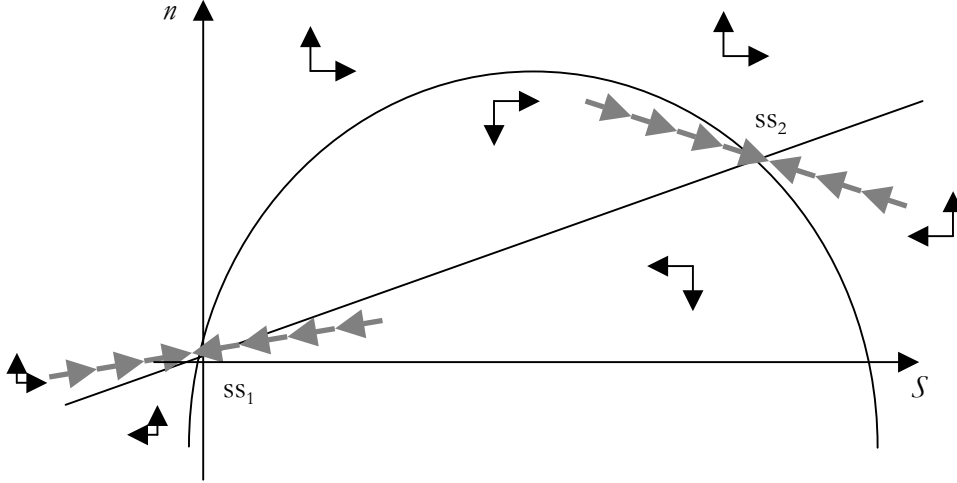
Analysing the case in which there is a positive flow of new entrants,  $\dot{n} > 0$ , we can see that, *ceteris paribus*, the higher the obsolescence effect  $\delta$ , the greater the flow of entrants<sup>4</sup>: this means that in those industries where the process of technological change is particularly turbulent there are more opportunities for new firms to bring new technological expertise into the industry and exploit the profit opportunities.

Drawing the phase lines of the two differential equations on the same graph, we can study the dynamic properties of the model (Figure 3).

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<sup>4</sup> The condition  $\dot{n} > 0$  implies  $\frac{1}{\phi} \left[ S r - \frac{(1 - \tau)(\alpha S - \beta n)}{\gamma} - g S^\theta + \delta S \right] > 0$ , that is:  
 $\delta S + r S > g S^\theta + \frac{\Pi}{\gamma}$ .

Figure 3: Representation of the dynamics of the system



As we can see diagrammatically, there are two saddle paths that lead to two stable steady states:  $ss_1$  and  $ss_2$ . The values of  $S$  and  $n$  in steady state  $ss_1$  are  $S^*=0$  and  $n^*=0$ .

In order to find the values of  $S$  and  $n$  in steady state  $ss_2$ , we need to solve the system given by (11) and (12). Equating the right-hand side of the two equations we have:

$$\frac{\alpha - \frac{r\gamma}{1-\tau}}{\beta} S = \frac{1}{(1-\tau)\beta} \{ \gamma g S^\theta - [r\gamma - (1-\tau)\alpha + \delta\gamma] S \}$$

which gives the value of  $S$  in the steady state:

$$(13) \quad S^* = \left[ \frac{g}{\delta} \right]^{\frac{1}{1-\theta}}$$

The corresponding value of  $n$  is:

$$(14) \quad n^* = \frac{\alpha - \frac{r\gamma}{1-\tau}}{\beta} \left[ \frac{g}{\delta} \right]^{\frac{1}{1-\theta}}$$

We also assume that the following condition holds:

$$(1-\tau)\alpha < r\gamma + \delta\gamma - \delta\gamma\theta$$

such that the steady state value  $S^*$  is greater than  $S_{max}$ .

Summing up the conditions we assume to hold, we have:

$$(15) \quad r\gamma < (1-\tau)\alpha < r\gamma + \delta\gamma - \delta\gamma\theta$$

### 3.2.2. Analysis of the Dynamics Towards the Steady State

We have seen that the model exhibits two stable saddle paths. This outcome leads immediately to our first result.

**PROPOSITION 1**

*In order for the system to converge towards the stable steady state characterised by positive values of the variables, we need the model to inherit (start with) an initial level of knowledge  $S$  and a number of starting firms of a certain positive magnitude. Otherwise, the system moves towards the origin and the industry implodes (steady state  $ss_1$ ).*

We interpret this result as follows: as necessary conditions for the birth of an industry, given the level of knowledge, we require the existence of a minimum number of starting firms, such that the subsequent developments cannot occur along the second transitional path, i.e. that to industrial implosion. This means that if the initial number of starting firms is not high enough to sustain the development and the growth of the economy, then the industry does not take off.

Let us turn to the transitional dynamics towards the steady-state characterised by positive values of the relevant variables. The movement towards the stable steady state  $ss_2$  is visible in a decreasing number of firms  $n$ . At the conclusion of this transitional process, the steady state value  $n^*$  is lower than the starting  $n$ , and there is an increase in the level of knowledge  $S$ . This aspects leads us to our second result.

**PROPOSITION 2**

*The industry becomes more concentrated and at the same time the level of competencies grows.*

In other words, the industry becomes more concentrated at the same time as it becomes richer in terms of technological competencies and expertise. The cost of entry being defined as in equation (1),  $C = \gamma S$ , and given that  $S$  increases, then, it follows that during the adjustment process of the industry towards its steady state, the level of barriers to entry (i.e. the cost of entry) increases until consequently, there is no new entry<sup>5</sup>. Moreover, during the adjustment period, in which the cost of entry is increasing, firms are making decreasing profits in order for the arbitrage condition (2) to hold in the steady state. This means that some firms exit the industry, and the number of firms is decreasing towards the steady state (note, in fact, that the steady state value of  $n^*$  is lower that the initial one,  $n$ ), as shown in Figure 4.

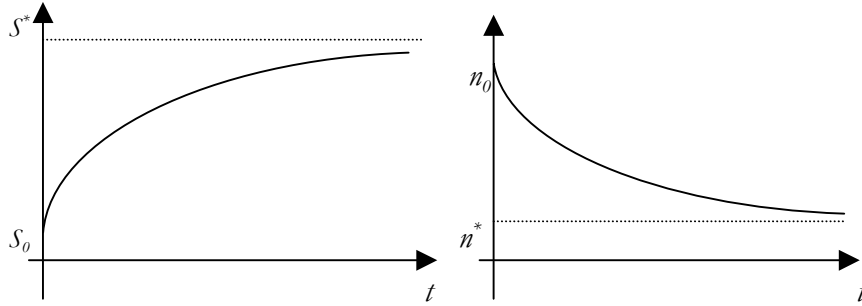
In the steady state, the level of technological knowledge is positively related to its growth rate  $g$  and inversely related to the rate of obsolescence  $\delta$ , as we expected. The number of firms in the steady state, also, is larger the greater the growth rate of technological knowledge and the lower the obsolescence rate are. Moreover, it is

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<sup>5</sup> Since  $C = \gamma S$ , then if  $\dot{S} > 0$  it follows that  $\dot{C} = \gamma\dot{S} + \gamma\dot{S} = \gamma\dot{S} > 0$

positively related to  $\alpha$ , the elasticity of profits to the level of knowledge (in the absence of taxation), and inversely related to  $\beta$ ,  $\gamma$  and  $r$ , that represent the elasticity of profits to the number of firms  $n$ , the cost of entry parameter, and the opportunity cost of funds, respectively.

Figure 4: Adjustment paths of  $S$  and  $n$



More precisely, the stable adjustment path (Figure 4) of the solution is described by:

$$(16) \quad \tilde{n}_t = n_t - n^* = [S_0 - S^*] \frac{1}{\mu_1 - a} b e^{\mu_1 t}$$

$$(17) \quad \tilde{S}_t = S_t - S^* = [S_0 - S^*] \frac{1}{\mu_1 - a} (\mu_1 - a) e^{\mu_1 t}$$

which, after simplifications, can be rewritten in matrix notation as follows<sup>6</sup>:

$$(18) \quad \begin{bmatrix} n_t \\ S_t \end{bmatrix} = \begin{bmatrix} n^* + [S_0 - S^*] \frac{1}{\mu_1 - a} b e^{\mu_1 t} \\ S^* + [S_0 - S^*] e^{\mu_1 t} \end{bmatrix}$$

#### 4. The Effects of Technological Change on Industrial Dynamics

The purpose of this section is to present the role of a change in the technological knowledge in the set-up of the model and consequently to analyse its results. As previously discussed, technological change has effects on the firms' existing knowledge. We model these effects, considering that such a shock could be characterised mainly by two features. First of all, the effect on the *type* of knowledge:

<sup>6</sup> See Appendix 3 for an alternative equivalent specification.

the existing type of knowledge could still be dominant in the industry in producing the standard product, or alternatively, as a consequence of the technological change, the current knowledge may no longer be required nor sufficient in order to efficiently produce. Secondly, as a consequence of the technological shock, the rate of *obsolescence* of the current knowledge might increase or decrease.

Following the discussion of the effects of technological change on the knowledge and competencies of the firms, we distinguish two cases, that we call: the case of competence-broadening and competence-narrowing technological change. The concepts of competence-broadening and competence-narrowing technological change, on the one hand, may resemble the theory of competence-enhancing and competence-destroying shocks, while, on the other, are different mainly because of their consideration of the explicit industrial applications (like products and variants of products) of the technological knowledge. Our concepts of competence-broadening and competence-narrowing technological change also relate to the various directions in which knowledge and capabilities can be advanced (Sutton, 2001). We have, then, a case (competence-broadening) in which alternative technological trajectories coexist and enlarge the availability of alternative types of products and technological applications, each of which has a positive share of total demand. On the other hand, we have a case of change in knowledge conditions (competence-narrowing) that leads to an enhance of capabilities only along a given trajectory, such that the stock of knowledge associated to alternative technological applications become obsolete and useless.

#### ***4.1. Competence-Broadening Technological Change***

We aim at representing a competence-broadening technological change in the set-up of the model by separating the two effects discussed above. Firstly, we consider that in the presence of a competence-broadening effect the current stock and type of knowledge is still required and necessary for producing efficiently and having a positive share of demand in the industry. In order to model this effect, let us consider the parameter  $a$ : in the absence of taxation, it indicates the elasticity of profits with respect to the current level of knowledge. If, following the technological change, the previously available competencies are still a necessary condition for efficiency (i.e. it is the dominant type of knowledge), then, we can impose the condition that parameter  $a$  is unaffected<sup>7</sup>. The second effect discussed above is modelled by a step change in the parameter  $\delta$ , which represents the obsolescence rate of the technological knowledge  $S$ . In this way, competence-broadening change has a positive effect on the existing stock of knowledge, by reducing the rate of obsolescence parameter pertaining to it. From now on, we assume that the changes in the parameters  $\alpha$  and  $\delta$  respect the assumption we stated in (15).

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<sup>7</sup> Alternatively, we might model an increase in  $a$ , but we study the case in which  $a$  is unaffected in order to keep the analysis as simple as possible.

Let us analyse the case of an unanticipated change occurring at time  $T$ . Let us examine the effects in the model by assuming, according to what we discussed above, that  $\alpha$  does not change,  $\bar{\alpha}$ , and that  $\delta$  decreases from  $\delta_1$  to  $\delta_2$ , with  $0 < \delta_2 < \delta_1$ .<sup>8</sup> The immediate effect of a step decrease in the obsolescence rate  $\delta$  is an upward shift of the  $\dot{n} = 0$  curve, while the  $\dot{S} = 0$  line is not affected. Consequently, there is a new saddle path passing through the new steady state  $ss_2$ !. The dynamics show that the

number of firms  $n$  jumps to the new saddle path with  $n_1^* = \frac{\alpha - \frac{r\gamma}{1-\tau}}{\beta} \left[ \frac{g}{\delta_1} \right]^{\frac{1}{1-\theta}}$  moving

$$\text{to } n_2^* = \frac{\alpha - \frac{r\gamma}{1-\tau}}{\beta} \left[ \frac{g}{\delta_2} \right]^{\frac{1}{1-\theta}}.$$

At time  $T$ , there is an immediate flow of new entrants into the industry,  $n_T$ . After this, the number of firms declines towards the new steady state, where the number of firms,  $n_2^*$ , is greater than the number in the old steady state  $n_1^*$ , but it is smaller than the number of firms existing in the industry immediately after the shock.

#### PROPOSITION 3

*After a competence-broadening technological change there is an overshooting process of the number of firms in the industry*

After the unanticipated change occurring at time  $T$ , we recognise two effects: an immediate positive flow of entry, provoked by the new opportunities that the competence-broadening shock creates; and a gradual process of adjustment towards the new equilibrium, which could be explained by an additional number of new entrants, and a process of exit by some of the existing firms or the new start-ups. The model, then, is able to predict the co-existence of entry and exit in the adjustment process towards the equilibrium after a technological shock.

As a consequence of the technological change, the level of the aggregate knowledge in the industry constantly grows from the starting level of the steady-state  $S_1^*$  to the

new one  $S_2^*$ , where:  $S_1^* = \left[ \frac{g}{\delta_1} \right]^{\frac{1}{1-\theta}}$  and  $S_2^* = \left[ \frac{g}{\delta_2} \right]^{\frac{1}{1-\theta}}$  (Figure 5). Clearly the increases

in the number of firms and in the level of knowledge of the industry are proportional to the decrease in the parameter  $\delta$ . The greater the change in the rate of obsolescence, the greater the increase in the number of firms  $n$  and in the stock of knowledge  $S$  of the new steady-state:

---

<sup>8</sup> Assuming that the parameter  $\delta_2$  is still greater than zero, the dynamic properties of the system (16), see condition (19), do not change.

$$n_2^* - n_1^* = \frac{\alpha - \frac{r\gamma}{1-\tau}}{\beta} g^{\frac{1}{1-\theta}} \left\{ \left[ \frac{1}{\delta_2} \right]^{\frac{1}{1-\theta}} - \left[ \frac{1}{\delta_1} \right]^{\frac{1}{1-\theta}} \right\}$$

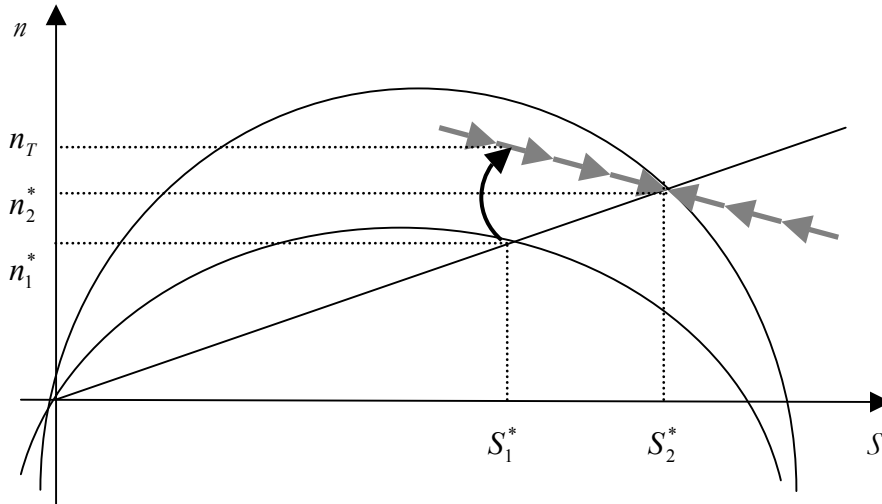
$$S_2^* - S_1^* = g^{\frac{1}{1-\theta}} \left\{ \left[ \frac{1}{\delta_2} \right]^{\frac{1}{1-\theta}} - \left[ \frac{1}{\delta_1} \right]^{\frac{1}{1-\theta}} \right\}$$

The transitional dynamics of  $n$  and  $S$  for  $t > T$  are given by:

$$(23) \quad n_t = n_2^* + [S_1^* - S_2^*] \frac{1}{\mu_1 - a} b e^{\mu_1(t-T)}$$

$$S_t = S_2^* + [S_1^* - S_2^*] e^{\mu_1(t-T)}$$

Figure 5: competence-broadening technological change:: the overshooting number of firms

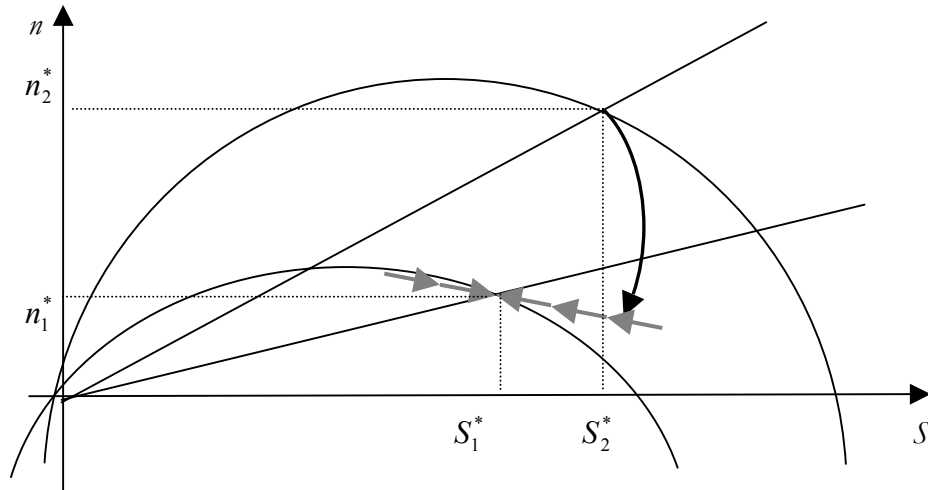


#### 4.2. Competence-Narrowing Technological Change

Let us consider now the consequences of a competence-narrowing technological change on the parameters of the model. First of all, the current stock of knowledge is negatively affected by the shock: the whole existing stock of competencies is no longer useful anymore in order to produce efficiently in the industry. After the competence-narrowing shock, we assume that most of the accumulated knowledge is superfluous and obsolete, and only a fraction of the existing type of knowledge is necessary and sufficient in order to produce the so called standard-dominant product

along the emerging superior particular trajectory. We model this aspect by assuming a decrease in the parameter  $\alpha$ . Being  $\alpha$  (in the absence of taxation) the elasticity of profits with respect to the current level of knowledge, a decrease in this parameter signifies the fact that part of the current knowledge is not “profitable” anymore. For simplicity, we analyse the case of a step shift from  $\alpha_0$  to  $\alpha_1$  (with  $\alpha_0 > \alpha_1 > 0$ ), occurring at period  $T_1$ . Similarly, the second effect is introduced as a step change of the obsolescence rate of the technological knowledge:  $\delta$ , jumps upwards as well, illustrating the negative effect of the competence-narrowing change on the existing stock of knowledge. The consequences on the loci  $\dot{n} = 0$  and  $\dot{S} = 0$ , after the step changes in the parameters, are represented in Figure 6: both curves shift downwards determining a new steady state  $ss_1'$ .

Figure 6: Competence-narrowing technological change



Alternatively, and more realistically, we may think about a gradual decrease in the usefulness of the existing stock of knowledge  $S_t$  by assuming that the parameters of interest,  $\alpha$  and  $\delta$  progressively change over time. We model these decreases according to the following specifications:

$$(24) \quad \dot{\delta} = -\kappa[\delta(t) - \delta_1]$$

and

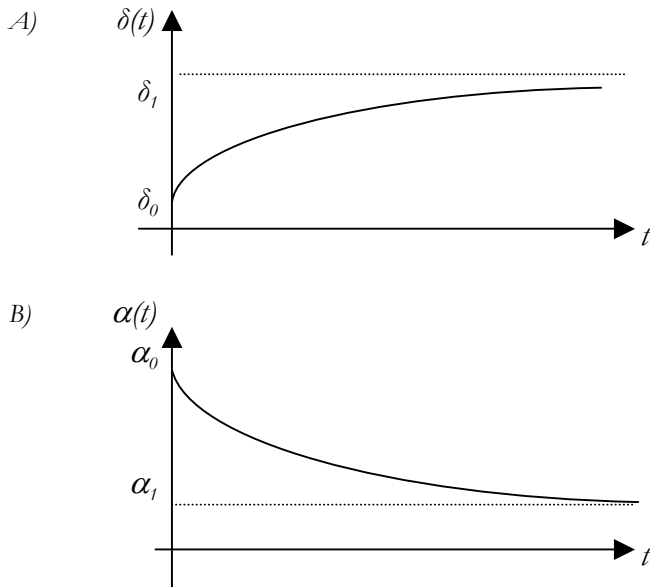
$$(25) \quad \dot{\alpha} = -\xi[\alpha(t) - \alpha_1]$$

Assume that the competence-narrowing shock occurs at time  $t_0$ . We have the initial conditions  $\alpha(0) = \alpha_0$ ,  $\delta(0) = \delta_0$ . Then,  $\delta_0$  is the obsolescence rate at the time of the shock, and  $\delta_1$  is the upper value of the parameter, with  $\delta_1 > \delta_0$ . Similarly,  $\alpha_0$  is the obsolescence rate at the time of the shock, and  $\alpha_1$  is the lower value of the parameter, with  $0 < \alpha_1 < \alpha_0$ .

The system, is now composed of four differential equations as follows:

$$(26) \quad \begin{cases} \dot{S} = \frac{\gamma r - (1-\tau)\alpha(t)}{\gamma} S + \frac{(1-\tau)\beta}{\gamma} n \\ \dot{n} = \frac{1}{\phi} \left[ \frac{\gamma r + \gamma \delta(t) - (1-\tau)\alpha(t)}{\gamma} S - gS^o + \frac{(1-\tau)\beta}{\gamma} n \right] \\ \dot{\delta} = -\kappa[\delta(t) - \delta_1] \\ \dot{\alpha} = -\xi[\alpha(t) - \alpha_1] \end{cases}$$

Figure 7: Time path of the parameters  $\delta$  and  $\alpha$ , in the presence of a competence-narrowing shock



The dynamics of the system are given by a continuous movement of the relevant variables, the number of firms  $n$  and the stock of competencies  $S$ , towards the new steady state values, given by:

$$(27) \quad n_1^* = \frac{\alpha_1 - \frac{r\gamma}{1-\tau}}{\beta} \left[ \frac{g}{\delta_1} \right]^{1-\theta}$$

and

$$(28) \quad S_1^* = \left[ \frac{g}{\delta_1} \right]^{1-\theta}$$

with the parameters varying from  $t_0$  to the time of arrival of the system at the new steady state according to:

$$(29) \quad \delta(t) = \delta_1 - (\delta_1 - \delta_0)e^{-\kappa t}$$

and

$$(30) \quad \alpha(t) = \alpha_1 + (\alpha_0 - \alpha_1)e^{-\xi t}$$

which are represented graphically in Figures 7A and 7B.

Visually, we could imagine the representation of the system drawn in Figure 6, moving also along a third axis, i.e. time, as the parameters approach the new values.

## 5. The Industry Life Cycle Story Reconsidered

Among the industrial dynamics models that explain industries' evolution, the so-called "industry life cycle" approach has gained much consensus both among theorists and empirical researchers. According to the industry life cycle models, the process of entry follows a given path that shows an increasing number of entering firms in the early stage of the industry evolution and then a subsequent decline of the entry rate, that leads the industry towards a more concentrated structure. The turning point of the evolutionary dynamics is found in the shakeout period, in which a massive number of firms exit the industry and the process of entry starts to stabilise, leading then to a lower total number of producers.

Klepper (1996) imputes the driving force of the shakeout phenomenon to the role of the economies of scale in the production process: bigger firms benefit more from a cost reduction because they produce a larger output, and consequently they have more incentive to invest in process R&D. This leads the less innovative and efficient firms to exit the market, given that the industry price falls over time as long as output expands. In this context, later entrants need to be relatively more efficient in product innovation in order to find entry profitable. However, at some point in time the entry process stops, given that the potential entrants have a larger and larger cumulative disadvantage with respect to incumbent firms. Klepper explicitly refutes the explanation proffered by other studies of the shakeout as a consequence of the

emergence of a dominant design. According to these scholars (Utterback and Abernathy, 1975; Abernathy and Utterback, 1978), in the early phase of the industry life cycle, many firms producing different variants of the product enter the industry, until a dominant design emerges. After this, buyers' preferences become stable and focused on given features of the product that constitute the dominant design: accordingly, opportunities for improving and altering the product start to deplete. Those firms who are able to produce this design prosper, while those firms unable to produce efficiently must exit the market. The emergence of a dominant design, then, gives birth to the shakeout period.

We propose in our model a different interpretation of the industry life cycle story. First of all, we argue that in order to have the birth of an industry, given the level of initial aggregated technological knowledge, we need a minimum number of starting firms (see Proposition 1). This result might explain why some industries did not take off. We claim that in order to study the infancy phase of the industry life cycle evolution, it is extremely important to look at the initial number of firms.

Secondly, we aim at proposing a different explanation of the industry life cycle evolution, referring to the concepts of competence-broadening and competence-narrowing technological change. We explicitly depart from Klepper's explanation in the sense that we do not refer to the existence of economies of scale as the cause of the emergence of a dominant firm (or a few big producers). We also depart, to a lesser extent, from Abernathy and Utterback's argument: we do not refer to the distinction between firms that "decide" to produce the dominant design and those who do not, while we focus more on the role of the technological knowledge and competencies of the existing firms. Recall that, in our model, we look at the aggregate number of firms and at the accumulated stock of technological knowledge. We claim that the industry life cycle evolution might be interpreted in accordance with the development of the technological knowledge of the relevant technology in the industry. In the discussion of our model, we have seen that in the presence of a change that improves upon the existing technological knowledge, new firms are stimulated to enter the industry and henceforth contribute to advancing the technological frontier. In the presence of a competence-narrowing change, on the other hand, the technological knowledge is eroded and only part of it survives the shock.

We claim that the emergence of a dominant design is comparable to the occurrence of a competence-narrowing change in the technological knowledge of the incumbent producers: when a dominant design emerges, the other variants of the product are not worthwhile any longer. Only a fraction of the existing stock of the technological knowledge survives the shock, and the other fraction becomes worthless and obsolete. Producers who do not embody or who are not able to acquire or develop the expertise and knowledge in order to produce efficiently along the dominant trajectory find their existing knowledge useless and eventually exit the market.

We claim that a possible pattern of industries' evolution might derive from successive periods of competence-broadening changes, in which new firms enter and the technological knowledge is variegated and improving leading to the coexistence of various types of products and applications, followed by a competence-narrowing change that defines the dominant technological trajectory and the dominant product in the market. As a consequence of the latter change, only part of the existing knowledge is revealed to be useful, and only those firms that are able to exploit the opportunities associated with it survive.

To put it in another way, we claim that our model might help in the understanding of the reasons why some industries do not follow the industry life cycle path. We argue that in those industries where the technological knowledge is constantly developing and evolving with regard to the existing stock of knowledge in the industry, and where different variants of the products and alternative different applications are available and required in the industry, then an industry life cycle dynamics is not likely to occur (see for example the laser industry and the "flowmeter" industry). According to this view, then, the phenomenon of the "shakeout" in the number of firms is explained as an exogenous event (see Jovanovic B., MacDonald G., 1994). In absence of this exogenous shock, then, the evolutionary path of the industrial structure is not likely to follow the industry life cycle predictions.

Moreover, our model leads to the interpretation of the "technological succession" theory as a series of successive periods of competence-broadening and competence-narrowing technological change that give rise to a technological life cycle evolution, in which different technologies, and different populations of firms associated with them, displace each other and improve with the contribution of new entrants in a dynamic fashion.

## **6. The Impact of Public Policy on Industrial Dynamics**

The concentration of an industry is clearly affected by the rate of new entry and the survival of incumbents and entrants. If we are interested, then, in the efficiency of the market and in its structure, entry is a relevant phenomenon to take into account when considering public industrial policies: by stimulating entry it is possible to affect market performance. In this section, we aim to study the effects of policy choices by the political authorities. In particular, we focus our attention on public policies aiming at improving market competition and we analyse different possibilities of intervention.

### ***6.1. Fiscal Policies***

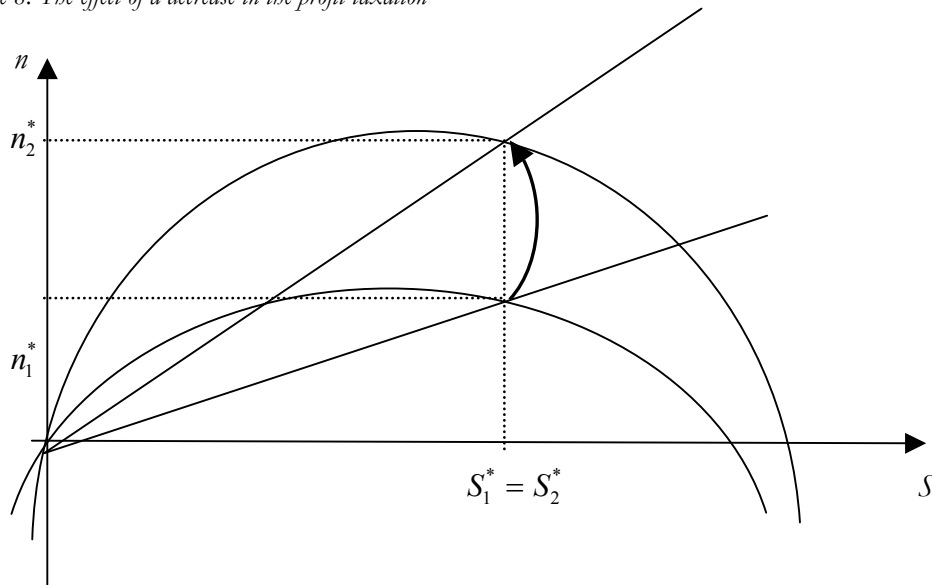
First of all, we consider the effects of a reduction in the profit tax rate on entry and on the evolution of knowledge  $S$ . The immediate effect of this intervention is an

increase in the level of profits for a given level of existing knowledge  $S$ . Let us assume that the tax rate  $\tau$  changes from  $\tau_1$  to  $\tau_2$  with  $\tau_1 > \tau_2$ . The consequence of this change on the dynamic system is given by a shift upwards of the loci  $\dot{n} = 0$  and  $\dot{S} = 0$ . In the new steady state the number of firms has increased from  $n_1^*$  to  $n_2^*$ , proportionately to the decrease in the tax rate ( $\tau_1 - \tau_2$ ):

$$(31) \quad n_2^* - n_1^* = \left[ \frac{\tau_1 - \tau_2}{(1 - \tau_1)(1 - \tau_2)} \right] \frac{r\gamma}{\beta} \left[ \frac{g}{\delta} \right]^{\frac{1}{1-\theta}}$$

while the level of knowledge is not affected by the policy. As shown in Figure 8, the effect of a decrease in the profit taxation is an immediate flow of new entrants into the industry.

Figure 8: The effect of a decrease in the profit taxation



The intuitive explanation of this result is that, after the reduction in  $\tau$ , firms find entry more profitable, such that the number of firms in the market immediately increases. The level of technological knowledge is not affected by the fiscal policy, and consequently the technological performance of the industry is unaffected: in this case, entry has to be considered as a phenomenon merely driven by the new profitability opportunities created by the fiscal policy. The opportunities for higher profits, for a given level of  $S$ , trigger an immediate increase in the number of firms, that causes the system to immediately jump to the new steady-state equilibrium.

Firms enter in order to exploit the new opportunities and there is no effect on the rate of change in  $\dot{n}$ , such that there is no effect on the evolution of knowledge. Firms do not bring more knowledge: they only need to acquire the existing level of  $S$ , i.e. to pay the cost of entry, in order to be able to take advantage of the new profits opportunities. In the model, with a given  $\alpha$ , when  $\tau$  decreases, the existing level of aggregate knowledge is sufficient to sustain a bigger number of firms in the industry: new entrants do not need to “create” new technological knowledge (in other terms, to differentiate themselves) in order to be profitable but they only need to acquire the required share  $\gamma$  of the existing  $S$ . In this sense, then, entry is seen as a re-equilibrating process towards the steady state: it is exclusively a profit-driven phenomenon, differently from what we found in the basic set-up of the model.

### **6.2. Rental Markets**

Public authorities might attempt to stimulate the technological knowledge and the technological performance in the industry by creating a favourable rental market that could encourage new entrants. For instance, it is possible to think of efforts in this direction entailing policies that create a reduction in the cost of entry, via the creation of favourable conditions in the rental market. This would translate, in terms of our model’s parameters, to a reduction of  $\gamma$ . Again, the consequences of such a choice would be only a jump in the number of firms: new entrants find it profitable to enter after the reduction of the entry cost. We do not register any effect on the level of  $S$ . Such a policy, then, only creates more “space” for new firms in the industry, without affecting the level of technological knowledge. We claim that tailored policies and interventions are needed in order to stimulate technological growth, as analysed in the next section.

### **6.3. Subsidising “Technological Knowledge” Development**

We have seen that a decrease in the taxation of profits has the effect of stimulating entry in order to exploit the new profit opportunities, improving in this way the market competition. In the following, we investigate whether it could be possible to stimulate the creation of technological knowledge in the industry. To this end, we introduce in the model a type of subsidy in the structural evolution of  $S$  and we analyse the consequences of this modification. In particular, we assume that the government reduces the tax rate from  $\tau_1$  to  $\tau_2$ , with  $\tau_1 > \tau_2$ , forcing the firms to invest the amount of saved profits into an R&D function. This function,  $R(I)$ , contributes to the development of technological knowledge  $S$ , where  $I$  indicates the amount of profits invested in R&D after the decrease in the tax rate, with:

$$(32) \quad I = \Pi_2 - \Pi_1 = (\tau_1 - \tau_2)(aS - \beta n)$$

We specify for simplicity a linear R&D function as follows:

$$(33) \quad R(I) = \eta(\tau_1 - \tau_2)(\alpha S - \beta n)$$

and we introduce this effect in the structural equation of  $S$ , obtaining:

$$(34) \quad \begin{aligned} \dot{S} &= gS^\theta - \delta S + \phi \dot{n} + R(I) = \\ &= gS^\theta - \delta S + \phi \dot{n} + \eta(\tau_1 - \tau_2)(\alpha S - \beta n) \end{aligned}$$

With this structural form, we are indirectly modelling a political intervention of entry sustenance. In facts, the de-taxed profits contribute to the creation of new technological knowledge, via the R&D function, and the creation of new knowledge stimulates a flow of new entrants.

The system, whose dynamics we want to investigate, then becomes:

$$(35) \quad \begin{cases} \dot{S} = \frac{\gamma r - (1 - \tau_1)\alpha}{\gamma} S + \frac{(1 - \tau_1)\beta}{\gamma} n \\ \dot{n} = \frac{1}{\phi} \left[ \frac{\gamma r + \gamma \delta - (1 - \tau_1)\alpha - \alpha \gamma \eta(\tau_1 - \tau_2)}{\gamma} S - gS^\theta + \frac{[(1 - \tau_1) + \gamma \eta(\tau_1 - \tau_2)]\beta}{\gamma} n \right] \end{cases}$$

The loci  $\dot{n} = 0$  and  $\dot{S} = 0$  are respectively:

$$n = \frac{1}{[(1 - \tau_1) + \gamma \eta(\tau_1 - \tau_2)]} \{ \gamma g S^\theta - [r\gamma - (1 - \tau_1)\alpha + \delta\gamma - \alpha \gamma \eta(\tau_1 - \tau_2)] S \}$$

and

$$n = \frac{\alpha - \frac{r\gamma}{1 - \tau_1}}{\beta} S$$

The resulting steady-state values are:

$$(36) \quad S^* = \left\{ \frac{(1 - \tau_1)g}{(1 - \tau_1)\delta - r\gamma \eta(\tau_1 - \tau_2)} \right\}^{\frac{1}{1-\theta}}$$

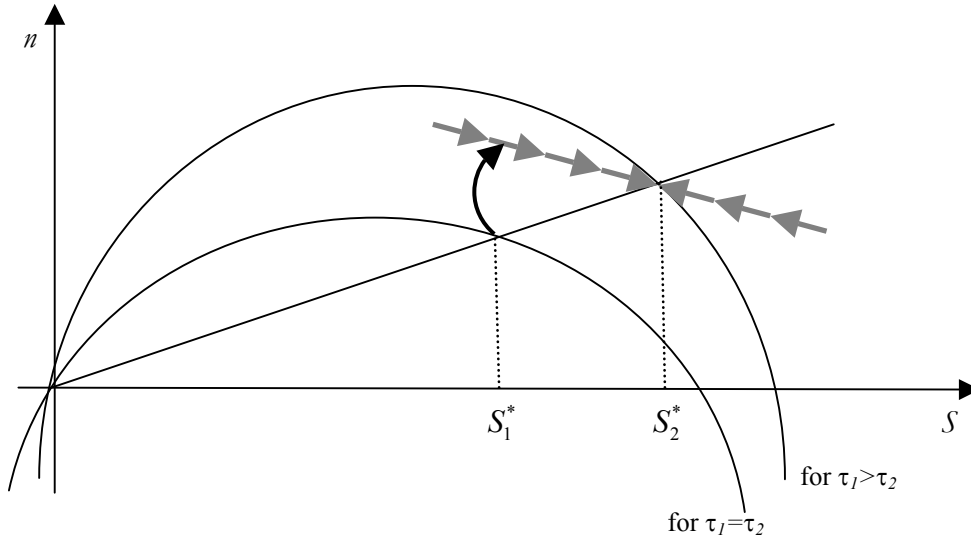
and

$$(37) \quad n^* = \frac{\alpha - \frac{r\gamma}{1 - \tau_1}}{\beta} \left\{ \frac{(1 - \tau_1)g}{(1 - \tau_1)\delta - r\gamma \eta(\tau_1 - \tau_2)} \right\}^{\frac{1}{1-\theta}}$$

If we consider a situation in which the economy is in the steady-state  $E_1$  with the tax rate equal to  $\tau_1$ , the effects of a policy intervention subsidising the creation of technological knowledge via a tax rate reduction from  $\tau_1$  to  $\tau_2$ , as we considered above, are the following: the number of firms increases, as well as the level of general

technological knowledge in the industry, which gradually moves towards the new steady-state equilibrium  $E_2$  (Figure 9).

Figure 9: Subsidising the creation of technological knowledge via a tax rate reduction



Our claim, then, is that in order to stimulate the development of technological competencies, fiscal policies by themselves are not sufficient, as we demonstrated in section 7.1. Political authorities also need to implement an intervention specifically tailored to R&D and knowledge creation: such a policy could be, for example, a de-taxation of R&D investments. In relation to this, the structure of the R&D function plays a key role, both from the perspective of purely economic outcomes and that of success of the political intervention: the more efficient the R&D function (i.e. the bigger  $\eta$ ), the higher the values of  $n$  and  $S$  in the steady state.

## 7. Agglomeration Effects on Industrial Dynamics

In this section, we explore a version of the model presented above in which the number (stock) of firms positively influences on the knowledge developments. We might interpret this formulation of the model as incorporating the presence of positive agglomeration effects in knowledge creation.

The relevance of the “economic geography” approach and of the “spatial knowledge spillover” effects has recently gained much attention from economics scholars, both from a theoretical and from an empirical point of view. The examination and comprehension of the results of the agglomeration factors also

shows its importance when we consider economic policies aiming at stimulate the regional development of certain areas of particular interest.

Since the last century, the significance of local effects has been recognised. Alfred Marshall stressed the role of the so-called “industrial atmosphere” in determining the diffusion of knowledge and the communication flows in a given area. The economic geography literature shows that the effectiveness of communication flows increases with the decrease of the distance between the two communicating agents. The importance of communication flows originating in a certain area, and the area’s absorptive capabilities regarding information generated by other actors, are strongly dependent on the existing level of knowledge and competencies of the area itself. The presence of an interdependence of agglomeration processes and high level of innovative activity is likely to be expected. Following this argument, according to the “technological infrastructure approach,” the innovative performance of a region is, explained by the presence of agglomeration of some factors like networks of firms, industrial and university R&D and the presence of business-service firms (Feldman, 1994).

In this section, we introduce an agglomeration effect in the model, reformulating the evolution equation of knowledge above (4) as follows:

$$(38) \quad \dot{S} = g S^\theta - \delta S + \phi \dot{n} + \rho n$$

where the parameter  $\rho$ , with  $\rho > 0$ , indicates a positive agglomeration effect of  $n$  on  $S$ . The goal of this section is to analyse the consequences of the presence of agglomeration effects on the evolution of the industry.

The dynamic conditions that govern the evolution of the system are now:

$$(39) \quad \dot{S} = S r - \frac{(1-\tau)(\alpha S - \beta n)}{\gamma}$$

and

$$(40) \quad \dot{n} = \frac{1}{\phi} \left[ S r - \frac{(1-\tau)(\alpha S - \beta n)}{\gamma} - g S^\theta + \delta S - \rho n \right]$$

and the corresponding loci  $\dot{S} = 0$  and  $\dot{n} = 0$  in the plane  $(S, n)$  are given respectively by the following conditions:

$$n = \frac{1}{(1-\tau)\beta - \gamma\rho} \{ \gamma g S^\theta - [r\gamma - (1-\tau)\alpha + \delta\gamma] S \}$$

and

$$n = \frac{\alpha - \frac{r\gamma}{1-\tau}}{\beta} S$$

The new steady-state values are as follows: first of all, we have again a steady state in which  $S^*=0$  and  $n^*=0$ ; secondly, we have a steady state  $ss_2$  in which

$$(41) \quad S^* = \left\{ \frac{(1-\tau)\beta g}{(1-\tau)\beta\delta - \rho[(1-\tau)\alpha - r\gamma]} \right\}^{\frac{1}{1-\theta}}$$

and

$$(42) \quad n^* = \frac{\alpha - \frac{r\gamma}{1-\tau}}{\beta} \left\{ \frac{(1-\tau)\beta g}{(1-\tau)\beta\delta - \rho[(1-\tau)\alpha - r\gamma]} \right\}^{\frac{1}{1-\theta}}$$

Since  $r\gamma < (1-\tau)\alpha$ , as we assumed in section 4.2.1, we rewrite the new steady-state value of  $S$  as follows:

$$(43) \quad S^* = \left\{ \frac{g}{\delta - \frac{\rho[(1-\tau)\alpha - r\gamma]}{(1-\tau)\beta}} \right\}^{\frac{1}{1-\theta}}$$

It is immediately clear that the new values of  $n$  and  $S$  are greater than the old steady-state values as a consequence of the presence of the agglomeration effect in the model, as we expected. In particular, it is possible to show that the greater the elasticity of  $S$  with respect to the parameter  $\rho$ , the greater the effect on the steady-state values of  $S$  and  $n$ . To show this, let us compute  $\frac{\partial S^*}{\partial \rho}$ :

$$\frac{\partial S^*}{\partial \rho} = \frac{1}{1-\theta} \left\{ \frac{-(1-\tau)\beta g [r\gamma - (1-\tau)\alpha]}{[\rho[r\gamma - (1-\tau)\alpha] + (1-\tau)\beta\delta]^2} \right\}^{\frac{\theta}{1-\theta}}$$

$$\text{Sign} \left( \frac{\partial S^*}{\partial \rho} \right) = \text{Sign} \{ -(1-\tau)\beta g [r\gamma - (1-\tau)\alpha] \}$$

which is clearly positive since  $r\gamma < (1-\tau)\alpha$ , as stated in expression (15).

From our investigation, it follows that in the presence of agglomeration effects, both the market performance, in terms of degree of competition, and the knowledge creation in the industry are improved. Our claim might be interesting for the public authority when considering local development policies. In particular, our conclusion supports the scholars that emphasise the importance of “pure agglomeration factors”<sup>9</sup>. From this point of view, then, interventions aiming at improving the

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<sup>9</sup> Theories of location benefits distinguish two parts: geographic benefits that are dependent on the location and independent from other agents’ activities, and agglomeration benefits that are dependent on the other agents but independent from the characteristics connected to a particular location.

infrastructure system, the communication flows among agents, as well as supporting entry and clusters development (for example, science parks), provide an effective stimulus to the economic system.

## 8. Conclusions

The paper presented a model of industrial evolution in a dynamic setting. We defined the evolution of the technological knowledge  $S$  in the industry and we related the changes in  $S$  to the contributions and advancements embodied in new entering firms,  $\dot{n}$ . The process of entry is thus defined as a competence-driven phenomenon. The results of the model show the transitional dynamics of both the stock of knowledge and the number of firms, and their steady-state values, which are positively related to the growth rate  $g$  and inversely related to the rate of obsolescence  $\delta$ . During the adjustment process, we show that the level of barriers to entry increases, the industry becomes more concentrated and at the same time it becomes richer in terms of technological knowledge and expertise.

We explored the effects of a technological change on the evolution of the industry, distinguishing between competence-broadening and competence-narrowing shocks. We then gave an explanation of the industry life cycle evolution in terms of succession of changes affecting the existing technological knowledge. We argue that the reasons why some industries do not follow the industry life cycle path might be explained by our model. In absence of an exogenous shock that leads to a competence-narrowing technological change, the evolutionary path of the industrial structure is not likely to follow the industry life cycle pattern.

The last part of the paper focused on fiscal policies and agglomeration effects. We show that fiscal policies by themselves are not sufficient to stimulate the development of technological competencies: political authorities need to focus on policies specifically tailored to R&D and knowledge creation, for example, a de-taxation of R&D investments. In order to provide an effective stimulus to the economy, the authorities may also intervene through investing on the infrastructure system, as the analysis of the agglomeration effects has shown.

## Appendix 1

Assume that the equation defining the evolution of knowledge is a logistic function. In this way, we are considering a growth process of  $S$  that is increasing at a faster rate at the beginning of its evolution. Equation (4) then becomes:

$$(4a) \quad \dot{S} = cS \left( 1 - \frac{S}{K} \right) - \delta S + \phi \dot{n}$$

where  $c$  is the speed of adjustment of  $S$  and  $K$  represents the “carrying capacity”. The dynamic system of differential equations is now given by:

$$(10a) \quad \begin{cases} \dot{S} = \frac{\gamma r - (1 - \tau)\alpha}{\gamma} S + \frac{(1 - \tau)\beta}{\gamma} n \\ \dot{n} = \frac{1}{\phi\gamma} \left\{ \gamma \frac{c}{K} S^2 + S [\gamma(r + \delta - c) - (1 - \tau)\alpha] + (1 - \tau)\beta n \right\} \end{cases}$$

In the steady state, the loci of points in the plane  $(S, n)$  associated to  $\dot{S} = 0$  does not change, while  $\dot{n} = 0$  gives now:

$$(12a) \quad n = \frac{1}{(1 - \tau)\beta} \left\{ S [(1 - \tau)\alpha - \gamma(r + \delta - c)] - \gamma \frac{c}{K} S^2 \right\}$$

which has a similar shape to equation (12), represented in Section 3.2.1. Graphically, in the  $\{S, n\}$  space, we have the intersections:

$$S_1 = 0$$

and

$$S_2 = \frac{K}{\gamma c} [(1 - \tau)\alpha - \gamma(r + \delta - c)]$$

and a maximum in:

$$S_{\max} = \frac{K}{2\gamma c} \frac{(1 - \tau)\alpha - \gamma(r + \delta - c)}{(1 - \tau)\beta}$$

The new steady state values are given by:

$$[n^* = 0; S^* = 0]$$

and

$$(13a) \quad \left[ n^* = \frac{\alpha - \frac{r\gamma}{1 - \tau}}{\beta} \cdot \frac{(c - \delta)K}{c}; S^* = \frac{(c - \delta)K}{c} \right]$$

Considering the only relevant steady state, then, the greater  $c$  and  $K$  and the lower the obsolescence effect  $\delta$ , the higher the number of firms in equilibrium as well as the higher the level of knowledge in the industry.

The condition we assume to hold, in order to have a negative eigenvalue and a positive one, as in expression (19), is that  $c > \delta$ : in this case both the steady states are again saddle-path stable.

The main results of our analysis are not affected by the selected specification.

## Appendix 2: Linearisation of the Model and Adjustment Path Towards the Equilibrium

Linearising the system given by the two first order differential equations:

$$(A2.1) \quad \begin{cases} \dot{S} = \frac{\gamma r - (1-\tau)\alpha}{\gamma} S + \frac{(1-\tau)\beta}{\gamma} n \\ \dot{n} = \frac{1}{\phi} \left[ \frac{\gamma r + \gamma \delta - (1-\tau)\alpha}{\gamma} S - g S^\theta + \frac{(1-\tau)\beta}{\gamma} n \right] \end{cases}$$

we are able to analyse the transitional dynamics toward the steady state and depict a phase diagram in the  $\{S, n\}$  space.

We can rewrite the system for simplicity as:

$$\begin{cases} \dot{n} = G(n, S) \\ \dot{S} = F(n, S) \end{cases}$$

We can compute:

$$\begin{aligned} (a) \quad & \frac{\partial G}{\partial n} = \frac{(1-\tau)\beta}{\phi\gamma} \\ (b) \quad & \frac{\partial G}{\partial S} = \frac{\gamma r + \gamma \delta - (1-\tau)\alpha}{\phi\gamma} - \frac{g\theta}{\phi} S^{\theta-1} \\ (c) \quad & \frac{\partial F}{\partial n} = \frac{(1-\tau)\beta}{\gamma} \\ (d) \quad & \frac{\partial F}{\partial S} = \frac{\gamma r - (1-\tau)\alpha}{\gamma} \end{aligned}$$

Let us indicate the deviations from the steady state as follows:

$$\tilde{S} = S - S^*$$

$$\tilde{n} = n - n^*$$

and the Taylor expansions of  $G$  and  $F$  as:

$$G(S, n) = G(S^*, n^*) + \tilde{n} \frac{\partial G(S^*, n^*)}{\partial n} + \tilde{S} \frac{\partial G(S^*, n^*)}{\partial S} + o(\tilde{S}, \tilde{n})$$

$$F(S, n) = F(S^*, n^*) + \tilde{n} \frac{\partial F(S^*, n^*)}{\partial n} + \tilde{S} \frac{\partial F(S^*, n^*)}{\partial S} + o(\tilde{S}, \tilde{n})$$

In the steady state we have  $F(S^*, n^*) = G(S^*, n^*) = 0$ , so we can write:

$$\dot{\tilde{n}} = \dot{n} - \dot{n}^* = \dot{n} \cong \tilde{n} \frac{\partial G(S^*, n^*)}{\partial n} + \tilde{S} \frac{\partial G(S^*, n^*)}{\partial S}$$

$$\dot{\tilde{S}} = \dot{S} - \dot{S}^* = \dot{S} \cong \tilde{n} \frac{\partial F(S^*, n^*)}{\partial n} + \tilde{S} \frac{\partial F(S^*, n^*)}{\partial S}$$

Rewriting this system in matrix form, we obtain:

$$\begin{bmatrix} \dot{\tilde{n}} \\ \dot{\tilde{S}} \end{bmatrix} = \begin{bmatrix} \frac{\partial G}{\partial n} & \frac{\partial G}{\partial S} \\ \frac{\partial F}{\partial n} & \frac{\partial F}{\partial S} \end{bmatrix} \begin{bmatrix} \tilde{n} \\ \tilde{S} \end{bmatrix}$$

where we call the matrix of the partial derivatives matrix  $M$ .

We can, then, write the linearised system as follows:

$$\begin{bmatrix} \dot{\tilde{n}} \\ \dot{\tilde{S}} \end{bmatrix} = \begin{bmatrix} \frac{(1-\tau)\beta}{\phi\gamma} & \frac{\gamma r + \gamma\delta - (1-\tau)\alpha}{\phi\gamma} - \frac{g\theta}{\phi} S^{*\theta-1} \\ \frac{(1-\tau)\beta}{\gamma} & \frac{\gamma r - (1-\tau)\alpha}{\gamma} \end{bmatrix} \begin{bmatrix} \tilde{n} \\ \tilde{S} \end{bmatrix}$$

or in system form:

$$\begin{cases} \dot{n} = \frac{(1-\tau)\beta}{\phi\gamma}(n - n^*) + \left[ \frac{\gamma r + \gamma\delta - (1-\tau)\alpha}{\phi\gamma} - \frac{g\theta}{\phi} S^{*\theta-1} \right] (S - S^*) \\ \dot{S} = \frac{(1-\tau)\beta}{\gamma}(n - n^*) + \frac{\gamma r - (1-\tau)\alpha}{\gamma} (S - S^*) \end{cases}$$

which in the neighbourhood of the steady-state  $ss_2$  takes the following form:

$$(A2.2) \quad \begin{cases} \dot{n} = \frac{(1-\tau)\beta}{\phi\gamma} n + \frac{\gamma r + \gamma\delta - (1-\tau)\alpha - \gamma\delta\theta}{\phi\gamma} S - \frac{\delta - g\theta}{\phi} \left[ \frac{g}{\delta} \right]^{\frac{1}{1-\theta}} \\ \dot{S} = \frac{(1-\tau)\beta}{\gamma} n + \frac{\gamma r - (1-\tau)\alpha}{\gamma} S \end{cases}$$

In order to study the dynamic solution of the system, we need to investigate the characteristic equation, that is given by:

$$\begin{vmatrix} \frac{(1-\tau)\beta}{\phi\gamma} - \mu & \frac{\gamma r + \gamma\delta - (1-\tau)\alpha}{\phi\gamma} - \frac{g\theta}{\phi} S^{*\theta-1} \\ \frac{(1-\tau)\beta}{\gamma} & \frac{\gamma r - (1-\tau)\alpha}{\gamma} - \mu \end{vmatrix} = 0$$

i.e.:

$$\mu^2 - \left[ \frac{(1-\tau)\beta}{\phi\gamma} + \frac{\gamma r - (1-\tau)\alpha}{\gamma} \right] \mu - \frac{(1-\tau)\beta}{\phi\gamma} \left[ \delta - g\theta S^{*\theta-1} \right] = 0$$

which gives the expression for the two eigenvalues:

$$(A2.3) \quad \mu_{1,2} = \frac{\left[ \frac{(1-\tau)\beta}{\phi\gamma} + \frac{\gamma r - (1-\tau)\alpha}{\gamma} \right] \pm \sqrt{\left[ \frac{(1-\tau)\beta}{\phi\gamma} + \frac{\gamma r - (1-\tau)\alpha}{\gamma} \right]^2 + 4 \frac{(1-\tau)\beta}{\phi\gamma} [\delta - g\theta S^{*\theta-1}]}}{2}$$

To determine the sign of the eigenvalues, we study the sign of the determinant of the matrix  $M$  of the partial derivatives defined above, that is:

$$4 \det(M) = -4 \frac{(1-\tau)\beta}{\phi\gamma} [\delta - g\theta S^{*\theta-1}]$$

In the steady state  $ss_1$ , we have  $S^*=0$ , so, since  $\delta > 0$ , the sign of the expression under examination is negative.

In the steady state  $ss_2$ , we have  $S^* = \left[ \frac{g}{\delta} \right]^{\frac{1}{1-\theta}}$ . After some manipulations we find:

$\delta = \frac{g}{S^{*1-\theta}}$  such that, substituting, we can rewrite the expression above as:

$$(A2.4) \quad \Delta = -4 \frac{(1-\tau)\beta}{\phi\gamma} \delta (1-\theta)$$

which is clearly negative, since  $\theta < 1$ .

It follows that, in both cases  $ss_1$  and  $ss_2$ , one eigenvalue is negative and one is positive (denoted  $\mu_1 < 0 < \mu_2$ ) such that both the steady states are saddle-path stable.

#### *Solution*

Let us rewrite the system generically as follows:

$$\begin{bmatrix} \dot{\tilde{n}} \\ \dot{\tilde{S}} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \tilde{n} \\ \tilde{S} \end{bmatrix}$$

An eigenvector associated with  $\mu_1$  is a vector  $[A \ B]'$  such that:

$$\mu_1 \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

which gives the system:

$$\begin{cases} \mu_1 A = aA + bB \\ \mu_1 B = cA + dB \end{cases}$$

But one of these equations is superfluous, because solving the system:  $P\underline{y} = \mu\underline{y}$ , for  $\underline{y} \neq 0$ , means  $(P - \mu)\underline{y} = 0$ , and so the matrix should have some linearly dependent rows. Then, we can take the equation:

$$\mu_1 A = aA + bB$$

i.e.

$$(\mu_1 - a)A = bB$$

that is satisfied by the values:

$$A = b$$

and

$$B = (\mu_1 - a)$$

In terms of our variables we have:

$$(A2.5) \quad \begin{aligned} A &= \frac{\gamma r + \gamma \delta - \gamma \delta \theta - (1 - \tau) \alpha}{\phi \gamma} \\ B &= \mu_1 - \frac{(1 - \tau) \beta}{\phi \gamma} \end{aligned}$$

These values represent one *particular solution* for the dynamic system considered.

The set of eigenvalues associated with  $\mu_1$  is the line (i.e. the vector space of dimension one):

$$\left\{ \text{for all } v \in \mathfrak{R}, \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} vb \\ v(\mu_1 - a) \end{bmatrix} \right\}$$

and a particular solution to the dynamic equations is given by:

$$\begin{bmatrix} \tilde{n}_t \\ \tilde{S}_t \end{bmatrix} = v_t \begin{bmatrix} b \\ \mu_1 - a \end{bmatrix}$$

which yields by replacement in the dynamic equations:

$$\begin{bmatrix} \dot{\tilde{n}}_t \\ \dot{\tilde{S}}_t \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \tilde{n}_t \\ \tilde{S}_t \end{bmatrix}$$

that is:

$$\dot{v}_t \begin{bmatrix} b \\ \mu_1 - a \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} v_t \begin{bmatrix} b \\ \mu_1 - a \end{bmatrix} = v_t \mu_1 \begin{bmatrix} b \\ \mu_1 - a \end{bmatrix}$$

which is summarised by:

$$\dot{v}_t = v_t \mu_1$$

i.e.

$$\frac{dv_t}{dt} = v_t \mu_1$$

Rewriting this expression as:

$$\frac{dv_t}{v_t} = \mu_1 dt$$

and integrating, gives:

$$\int_{v_0}^{v_t} \frac{dv_r}{v_r} = \int_0^t \mu_1 dr$$

that is:

$$\left| \ln(v_r) \right|_{v_0}^{v_t} = \mu_1 t$$

which finally gives:

$$v_t = v_0 e^{\mu_1 t}$$

Hence, this particular solution is of the type:

$$(A2.6) \quad \begin{bmatrix} \tilde{n}_t \\ \tilde{S}_t \end{bmatrix} = v_0 e^{\mu_1 t} \begin{bmatrix} b \\ \mu_1 - a \end{bmatrix}$$

which is the converging solution associated with the negative eigenvalue  $\mu_1$ .

It can be shown that we can proceed likewise for the positive eigenvalue  $\mu_2$ , and we can find the explosive solution associated with it.

The *general solution* is a linear combination of the two particular solutions. Since the second particular solution is explosive, the saddle-path includes only the first one. The initial value  $v_0$  is determined by the initial level of knowledge  $S$ . We have:

$$\tilde{S}_t = S_t - S^* = v_t (\mu_1 - a)$$

In  $t=0$ :

$$S_0 - S^* = v_0 (\mu_1 - a)$$

which, solving for  $v_0$ , gives:

$$v_0 = [S_0 - S^*] \frac{1}{\mu_1 - a}$$

In conclusion, the stable adjustment path (see Figure 4 in text) of the solution is described by:

$$\tilde{n}_t = n_t - n^* = [S_0 - S^*] \frac{1}{\mu_1 - a} b e^{\mu_1 t}$$

$$\tilde{S}_t = S_t - S^* = [S_0 - S^*] \frac{1}{\mu_1 - a} (\mu_1 - a) e^{\mu_1 t}$$

which, after simplifications, can be rewritten in matrix notation as follows<sup>10</sup>:

$$(A2.7) \quad \begin{bmatrix} n_t \\ S_t \end{bmatrix} = \begin{bmatrix} n^* + [S_0 - S^*] \frac{1}{\mu_1 - a} b e^{\mu_1 t} \\ S^* + [S_0 - S^*] e^{\mu_1 t} \end{bmatrix}$$

where  $\mu_1$ ,  $a$  and  $b$  are given by the expressions above.

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<sup>10</sup> See Appendix 3 for an alternative equivalent specification.

### Appendix 3

We have shown in section 4.3. that the stable adjustment paths towards the steady-state of  $n$  and  $S$  take the following form:

$$(A1) \quad \begin{bmatrix} n_t \\ S_t \end{bmatrix} = \begin{bmatrix} n^* + [S_0 - S^*] \frac{1}{\mu_1 - a} b e^{\mu_1 t} \\ S^* + [S_0 - S^*] e^{\mu_1 t} \end{bmatrix}$$

An alternative equivalent specification of the expression above is the following:

$$(A2) \quad \begin{bmatrix} n_t \\ S_t \end{bmatrix} = \begin{bmatrix} n^* + [n_0 - n^*] e^{\mu_1 t} \\ S^* + [S_0 - S^*] e^{\mu_1 t} \end{bmatrix}$$

In fact, following Romer (1996), the Taylor approximation around the steady-state could be expressed as the expression given in section 4.3.:

$$\begin{cases} \dot{n} = \dot{n} - \dot{n}^* = \frac{(1-\tau)\beta}{\phi\gamma} (n - n^*) + \left[ \frac{\gamma r + \gamma \delta - (1-\tau)\alpha - \gamma \delta \theta}{\phi\gamma} \right] (S - S^*) \\ \dot{S} = \dot{S} - \dot{S}^* = \frac{(1-\tau)\beta}{\gamma} (n - n^*) + \frac{\gamma r - (1-\tau)\alpha}{\gamma} (S - S^*) \end{cases}$$

It follows:

$$\begin{cases} \frac{\dot{n} - \dot{n}^*}{n - n^*} = \frac{(1-\tau)\beta}{\phi\gamma} + \left[ \frac{\gamma r + \gamma \delta - (1-\tau)\alpha - \gamma \delta \theta}{\phi\gamma} \right] \frac{S - S^*}{n - n^*} \\ \frac{\dot{S} - \dot{S}^*}{S - S^*} = \frac{(1-\tau)\beta}{\gamma} \frac{n - n^*}{S - S^*} + \frac{\gamma r - (1-\tau)\alpha}{\gamma} \end{cases}$$

Let us indicate:

$$\frac{\dot{n} - \dot{n}^*}{n - n^*} = \mu$$

Then, from the first equation of the system we have:

$$\mu = \frac{(1-\tau)\beta}{\phi\gamma} + \left[ \frac{\gamma r + \gamma \delta - (1-\tau)\alpha - \gamma \delta \theta}{\phi\gamma} \right] \frac{S - S^*}{n - n^*}$$

which we can rewrite as:

$$\mu = a + b \frac{S - S^*}{n - n^*}$$

where:

$$a = \frac{(1-\tau)\beta}{\phi\gamma}$$

and

$$b = \frac{\gamma r + \gamma \delta - (1 - \tau) \alpha - \gamma \delta \theta}{\phi \gamma}$$

as we indicated in section 4.3.

We can rewrite the expression above as:

$$S - S^* = \frac{(\mu - a)(n - n^*)}{b}$$

which we can substitute in the system (A1) obtaining the system (A2) as we claimed.

## References

- Abernathy W., Utterback J. (1978), "Patterns of Innovation in Industry", *Technology Review*, Vol. 80, No. 7, 40-47
- Aloi M., Dixon H. (2001), "Entry Dynamics, Capacity Utilisation, and Productivity in a Dynamic Open Economy", Discussion Paper No. 01/09, *University of Nottingham*
- Datta B., Dixon H. (2002), "Technological Change, Entry and Stock Market Dynamics: an Analysis of Transition in a Monopolistic Economy", *American Economic Review, Papers and Proceedings*, 92, 231-235
- Feldman M. (1994), "The University and Economic Development: the Case of Johns Hopkins University and Baltimore", *Economic Development Quarterly*, 8, 1
- Freeman C., Soete L. (1997), *The Economics of Industrial Innovation*, Third Edition, Pinter
- Jovanovic B., MacDonald G. (1994), "The Life Cycle of a Competitive Industry", *Journal of Political Economy*, Volume 102, 322-347
- Klepper S. (1996), "Entry, Exit, Growth and Innovation Over the Product Life Cycle", *American Economic Review*, Volume 86, 562-583
- Klepper S. (1997), "Industry Life Cycles", *Industrial and Corporate Change*, Volume 6, Number 8, 145-181
- Maggioni M. (2002), *Clustering Dynamics and the Location of High-Tech-Firms*, Physica-Verlag
- Polo M. (1993), *Teoria dell'Oligopolio*, Il Mulino
- Romer D. (1996), *Advanced Macroeconomics*, McGraw-Hill, New York
- Schumpeter J. (1934, first published 1911), *The Theory of Economic Development*, Harvard University Press, Cambridge Mass
- Schumpeter J. (1939), *Business Cycles*, MacGraw Hill, New York
- Schumpeter J. (1950, first published 1942), *Capitalism, Socialism and Democracy*, Harper Brothers, New York
- Sutton J. (2001), "Rich Trades, Scarce Capabilities: Industrial Development Re-visited", Discussion Paper No. EI/28, September 2001, London School of Economics and Political Science

- Tellis G., Golder P. (1996), "First to Market, First to Fail? Real Causes of Enduring Market Leadership", *Sloan Management Review*, Winter, 65-75
- Tushman M., Anderson P. (1986), "Technological Discontinuities and Organizational Environments", *Administrative Science Quarterly*, 31, 439-465
- Utterback J. (1994), *Mastering the Dynamics of Innovation*, Boston M.A., Harvard Business School Press
- Utterback J., Abernathy W. (1975), "A Dynamic Model of Process and Product Innovation", *Omega*, 3, 639-656
- Utterback J., Suarez F. (1993), "Innovation, Competition, and Industry Structure", *Research Policy*, Volume 22, 1-21
- Williamson O. (1975), *Markets and Hierarchies: Analysis and Antitrust Implications*, The Free Press, New York
- Windrum P. (1999), "Unlocking a Lock-in: Towards a Model of Technological Succession", *MERIT Research Memoranda*, Publication Number 99-010, University of Maastricht (<http://www-edocs.unimaas.nl/files/mer99010.pdf>)
- Windrum P., Birchenall C. (1998), "Is Product Life Cycle Theory a Special Case? Dominant Designs and the Emergence of Market Niches Through Coevolutionary-learning", *Structural Change and Economic Dynamics*, 9, 109-134
- Winter S. (1984), "Schumpeterian Competition in Alternative Technological Regimes", *Journal of Economic Behaviour and Organisation*, 5
- Winter S., Kaniovski Y., Dosi G. (2000), "Modeling Industrial Dynamics with Innovative Entrants", *Structural Change and Economic Dynamics*, 11, 255-293